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Dr. GREGORY's
ELEMENTS
OF
CATOPTRICS
AND
DIOPTRICS.

Tranſlated from the *Latin Original*,
With a large SUPPLEMENT,
By WILLIAM BROWNE, *M.D.*

THE SECOND EDITION.

To which is added,

An APPENDIX,

By J. T. DESAGULIERS, LL. D. F. R. S.

CONTAINING,

An Account of the REFLECTING TELESCOPES; and of the Inventors, Improvers, and Imitators of them, till they were brought to Perfection by JOHN HADLEY, Eſq; Vice-Preſident of the *Royal Society*.

With Original LETTERS which paſſed between Sir ISAAC NEWTON and Dr. JAMES GREGORY, relating thereunto.

NOW FIRST PUBLISHED.

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PREFACE

(To this SECOND EDITION)

By Dr. *DESAGULIERS.*

IT is almoſt twenty Years, ſince Dr. BROWNE deſired me to look over his Tranſlation of this Treatiſe of *Catoptrics* and *Dioptrics*, and his own large *Supplement*: I found the Tranſlation to agree with the Original, and the *Addenda* very proper for ſuch Perſons as apply themſelves to *Optics*; and accordingly allowed him to mention my Approbation of the Work.

Now the Book being out of Print, and a great Demand made for it, the Doctor was requeſted, by Mr. CURLL, to publiſh a new Edition of it: But as he lives at *Lynn* in *Norfolk*, he could not, as he wrote him Word, fix any certain Time for the Publication; he therefore has deſired me to do it for him, allowing me to make ſuch Emendations and Additions as I ſhould think fit. I find no Occaſion for *Emendations*, except a few Errors of the Preſs, and in the Plates, which I have corrected: But I have made ſome *Additions* in his *Introduction* and his *Supplement*; and have, by way of *Appendix*, given a full Deſcription of the two Sorts of *Reflecting* TELESCOPES, at preſent ſo much in Vogue, and ſo juſtly valued; which I have explained by ſeveral Figures in a *fourth* Plate.

The Reader will find who have been the Inventors and Imitators of theſe *Teleſcopes*, by Sir ISAAC NEWTON's and Dr. JAMES GREGORY's* LETTERS, written near ſixty Years ago; ſo much of which I have tranſcribed from the Originals (now in the Poſſeſſion of WILLIAM JONES, Eſq;) as relates to theſe Matters: And the Public are wholly indebted for the Uſe of them to that excellent Mathematician and ingenious Mechanic JOHN HADLEY, Eſq; Vice-Preſident of the *Royal Society*.

As it may be expected that I ſhould give ſome Account of our Author, I have here ſubjoined what ANTHONY *à* WOOD ſays of him (in his *Faſti Oxon.* Vol. II. Page 225.) *viz.*

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* Uncle to our Author, who wrote the *Optica promota*, and publiſhed it in 1663.

“ David Gregory, M. A. of the Univerſity of *Edinburgh*, was admitted to the Rectory of *Brightwell*, near *Wallingford* in *Berkſhire*, 1691. The ſame Year he accumulated the Degrees in Phyſic, and became a Maſter-Commoner of *Baliol-College*.

“ This Gentleman, who was born at *Aberdeen*, and moſtly educated there, hath extant, *Exercitatio Geometrica de Dimenſione Figurarum*; *ſive*, *Specimen Methodi generalis dimetiendi quaſdam Figuras*. *Edenb.* 1684 *qu.* At which time he was Mathematic Profeſſor there.

“ His Uncle, Mr. James Gregory, printed at *Padua*, in the Year 1667, a Book entituled, *Vera Circuli & Hyperbolæ Quadratura*.

“ In the *Philoſophical Tranſactions*, N°. 207, *Jan.* 1693, is, *Solutio problematica Florentini de veliformi quadrabili*, by our Author David Gregory, then Fellow of the *Royal Society*.

“ In the *Philoſophical Tranſactions*, N°. 214, is, *An Epiſtle from Dr.* Gregory

" *to Sir* Robbrt Southwell, aſ-
" certaining ſome *Mathematical Inven-*
" *tions* to their Authors. Dated *Nov.* 15,
" 1694.

" He publiſhed alſo, Catoptricæ
" *&* Dioptricæ *Sphericæ Elementa.*
" *Oxon.* 1695. 8vo." A Tranſlation of which being what is here undertaken. And thereunto are added,

I. A Method for finding the *Foci* of all *Specula*, as well as *Lens's*, univerſally. As alſo for *Magnifying* or *Leſſening* a given *Object* by a given *Speculum* or *Lens*, in any aſſigned Proportion, *&c.*

II. A *Solution* of thoſe *Problems* which Dr. Gregory has left undemonſtrated.

III. A particular Account of Microscopes and Telescopes, from Mr. Huygens. With an *Introduction*, ſhewing the Diſcoveries made by Catoptrics and Dioptrics. By Dr. Browne.

A

A LETTER *from Dr.* BROWNE *to Mr.* CURLL.

SIR,

Lynn, July 6, 1734.

I Received yours: GREGORY was the Child of my Youth; you was ſo good as to take it off my Hands, and undertook to bring it up and maintain it at your own Charge: So that it is now more properly yours than mine. As to a new Edition of it, I cannot engage to do it by any particular Time, becauſe our leiſure Hours are ſo very uncertain: So that if you could get the ſame thing done by the Gentlemen you mention (Dr. DESAGULIERS, or Mr. JONES) which muſt be worth your while, they are ſo much more capable of doing it than my ſelf, that, with the Recommendation of their Names, I apprehend the Work would meet with a much more favourable Reception from the Public; and ſecure, in conſequence, a larger Share of what you may expect from it your ſelf. Very ſure I am, that your publiſhing only a correct Edition of the ſelf-ſame Work, without Alterations and Augmentations, will never anſwer in any reſpect. But judge for your ſelf; and whatever it is you undertake, I heartily wiſh Succeſs; only, if you will take my Advice, don't grudge the compleating it on the beſt Terms you can, by either of the Gentlemen you named. I am

Your old Friend, and
Humble Servant,

W. BROWNE.

A Second LETTER *from Dr.* BROWNE *to Mr.* CURLL.

Dear SIR,

Lynn, Nov. 2, 1734.

I AM glad you have put Dr. GREGORY into ſo much better Hands than mine. I have nothing to add; nor would I by *any means*, by *any thing of mine*, diminiſh the Value of an Edition, which will be much more eſteemed, in *being republiſhed by Dr.* DESAGULIERS.

I am,

(With Service to Him)

His and your humble Servant,

W. BROWNE.

Dr. BROWNE's Two LETTERS to Mr. CURLL, ſhew that every thing has been done in Concert with That Gentleman.

J. T. DESAGULIERS.

Channel-Row,
Nov. 21, 1734.

PREFACE.

THESE *Elements of* Catoptrics *and* Dioptrics, *which were eleven Years ago read publickly in Lectures, in the Univerſity of* Edinborough (1684) *I have compoſed for the Uſe of young Students, in ſuch a manner, that nothing but* EUCLID'*s Geometry is required towards the underſtanding them. For though I have likewiſe demonſtrated from higher Principles, why Spheres and Conoids obſerve the ſame Laws, both in reflecting and refracting Light; yet thoſe who are ſolicitous only about the Properties of Plane and Sperical Surfaces, may, without the leaſt Inconveniency, paſs over all that. Theſe laſt are what we have more eſpecially conſidered, as alſo ſuch Optical Inſtruments as are made by a Combina-*

Combination of them; that is, whoſe Effects ariſe either from a ſingle Lens *or* Speculum, *or from ſeveral combined together. I have, as* KEPLER *did before me, made uſe of ſome* Poſtulata, *that come not quite up to Geometrical Strictneſs, but yet are of great Service in reſolving Queſtions in Natural Philoſophy, which would otherwiſe be extremely intricate. If theſe Elements be found capable of inſtructing ſuch as are leſs converſant in* Optics, I *ſhall have my* End.

Oxon. 1695

INTRODUCTION.

THOSE Queſtions concerning the Nature of LUCID BODIES, and of LIGHT, which uſually coſt Philoſophical Writers ſo much Pains and Trouble, we have, after the Example of Mathematicians, omitted. For if they, who by their Inventions, have ſo much improved this Science, had employed all their Time in enquiring into the abſolute Nature of its Object, and the moſt hidden Cauſes of its *Phænomena*, not contented with deducing after a Geometrical Manner from thoſe more ſimple and eaſily obſerved Properties of Light, others leſs obvious; Optics had fallen much ſhort of that Perfection to which they are now arrived. Therefore whether Light be the Action of the lucid Body driving on thoſe Bodies that lie next it, which likewiſe drive on others next to them, and ſo on of the reſt, none of them in the mean time ſingly moving any conſiderable Space; or whether it conſiſts, which is much more likely, of Corpuſcles projected with a very great Velocity from the lucid Body thro' the circumambient Spaces; or whether it be of a quite different Nature, and ſuch as may hereafter, or perhaps never, be perfectly diſcovered; yet we may eaſily be allowed to aſſume this Property of it, which is ſimple enough, and confirmed by Experiments, That from every lucid Point Rays are every way propagated in an Orb, and, in a Medium that is homogeneal, are diffuſed in right Lines (ſuch being the *ſhorteſt*) after the ſame uniform Tenour.

But if thoſe Rays meet with a Medium differently affected, whoſe Parts either ſtrike them back, or diffuſe them more or leſs than the Parts of the former Medium did, they will then ſuffer an *Inflexion*, by which general Name, I would, with other Authors, underſtand their Reflexion, as well as Refraction. For Light ſtriking upon a Surface, that abſolutely denies it Entrance, but yet hinders not its being diffuſed after the ſame manner as before, will all of it return back the eaſieſt way it can find, diffuſing itſelf ſtill as at firſt; this is called *Reflexion* of Light, and the Science which treats of the Laws it obſerves, according to the different Incidence of Rays upon Bodies of different Figures, is called *Catoptrics*. But if the Medium, upon which the Light ſtrikes, allows indeed a Paſſage to its Rays, but then ſo as that they muſt be either more or leſs diffuſed than before, every Ray will be inflected from the right Line, in which it was before diſpoſed to proceed, and this Inflexion is called *Refraction*; and the Science which demonſtrates the Laws and Effects of it, is called *Dioptrics.*

The *Radiant* is that, from every Point of which Rays are propagated.

Parallel Rays, are ſuch as are equi-diſtant from one another.

Diverging Rays, are ſuch, as if produced both ways, meet on the Side contrary to that towards which they move.

Converging Rays, are ſuch as, if produced, meet on the ſame Side towards which they move.

It muſt be obſerved, that this *Paralleliſm*, *Divergency*, and *Convergency*, is to be underſtood of Rays proceeding all from the ſame Point.

The *Focus* is that Point, in which Rays proceeding from the ſame Point of the Radiant, being produced, do meet; whence the *Focus* of parallel Rays is looked upon as infinitely diſtant.

The *Angle of Incidence*, is that which is made by the incident Ray, and a right Line perpendicular to the inflecting Surface at the Point of Incidence.

The *Angle of Reflexion*, is that which is made by the reflected Ray, and the ſame Perpendicular.

The *Angle of Refraction*, is that which is made by the refracted Ray, and the ſame Perpendicular produced.

The two following Propoſitions we have aſſumed for *Axioms*, becauſe they agree both with Geometrical Reaſoning and Experiments.

AXIOMS.

1. A Ray of Light falling perpendicularly upon an inflecting Surface, either proceeds directly forward, or is reflected back upon itſelf. For ſince the Direction of the Ray to the inflecting Surface is, of all that can be drawn from the radiant Point, either the *leaſt*, if the inflecting Surface be a Plane, or perhaps, where this Circumſtance is wanting, the *greateſt*, and in both Caſes a determinate, and *only one* of its kind; the Ray will ſtill perſiſt in

the ſame Direction, either proceeding forward, or returning backward. For there are innumerable right Lines inclined to this *only one* in any given Angle, no one of which can conſequently claim to its ſelf the Direction of the Ray with greater Juſtice than the reſt.

2. If a Plane be ſuppoſed, produced thro' the incident Ray, and a right Line perpendicular to the inflecting Surface at the Point of Incidence, the inflected Ray will likewiſe be found in the ſame Plane; or, which is the ſame thing, every Inflexion is made in a Surface that is perpendicular to the inflecting Surface. And this Surface ſhall, according to ALHAZEN, be called the *Plane of Inflexion.* For ſince this Plane is either the *leaſt* or *greateſt* of all that can be produced thro' the radiant Point, to the inflecting Surface, and conſequently an *only one*, the Propoſition is demonſtrated after the ſame manner as the former: And indeed if we more cloſely conſider it, we ſhall find the former to be only a particular Caſe of this latter.

INTRODUCTION;

Shewing the *Diſcoveries* made by

CATOPTRICS and DIOPTRICS.

By Dr. *BROWNE*.

IT is a great Encouragement for thoſe who would take the Pains to perfect themſelves in any Science, to be firſt informed of what Service it will be to underſtand it: And ſince there is no Part of Learning of ſo real and general Benefit to Mankind, as this of *Catoptrics* and *Dioptrics*, it is but Juſtice both to the Subject and Reader, to give ſome Account of the many wonderful Diſcoveries which we owe entirely to this Science.

The *Sight* of Man is of it ſelf confined to very narrow Views; and though it takes in a great part of the Creation at once, yet all is repreſented in Miniature and imperfectly. The naked *Eye* ſees only ſo much of external *Objects* as is ſufficient to move the principal Paſſions, and give notice of what more immediately concerns the Safety and Happineſs of the Animal. What is more than this, was left as a Subject for our Curioſity, upon which we might exerciſe thoſe Faculties which are beſtowed by our bountiful Creator for this very End, of ſearching into the aſtoniſhing Mechaniſm of all his Works, and from thence enlarging our Idea of his Greatneſs. *Objects* placed at a great Diſtance, whether upon the Surface of our Earth, or in the Heavens, are ſeen under ſo ſmall an Angle, that their Parts are not to be diſtinguiſhed one from another; and by this means thoſe diſtant charming Scenes of Nature were hid from us, which the Study of *Catoptrics* and *Dioptrics* has ſince laid open to our View. This noble Part of Knowledge teaches us how, by a due Poſition of *Glaſſes* ground into certain Figures, we may enlarge the Diameters of the heavenly Bodies, and all ſuch *Objects* to which we are allowed no nearer Approach, in what Proportion we pleaſe, and view them as perfectly and diſtinctly as if we could ſummon them before us, and command them to the End of our *Teleſcope*. This has brought us into a perfect Acquaintance with thoſe ſurprizing Parts of the Creation, which are far ſeparate from this Globe of ours, and with which we are allowed no Commerce but Looking. We can now perceive the *Sun* to be a vaſt Globe of Fire, and by the different *Phaſes* of all the *Planets*, that *he* is the Fountain of all their *Light*. The Surfaces of moſt of them appear like ſo many *Maps* of Land and Water, and there are few now but allow both them and the *fixed Stars* ſome nobler Uſe than to twinkle upon us o'Nights. By fixing upon ſome remarkable *Spots* on their Surfaces, and obſerving how they ſhift their Poſition, and in what time they again return to the ſame Place, we determine the Motion of theſe Bodies round their *Axes*, and the Time in which that Revolution is performed. Several *ſecondary Planets*, or *Satellites*, which were too ſmall for the naked *Eye*, are now diſcerned to move round *Jupiter* and *Saturn*, as the *Moon* round our Earth; and about the laſt of them is ſeen the particular *Phænomenon* of an *Annulus*, or *Ring*. Nor is the Diſcovery of theſe *Satellites* merely ſpeculative, but of prodigious Uſe and Advantage; for their *Eclipſes* have determined the *Velocity* of *Light*, as *Romer* has ſhewn,* and are ſo frequent, as to be the moſt conſtant Appearance the Heavens afford us at preſent, for the Solution of the great and valuable Problem of the *Longitude.* The Diſtances, Magnitudes, and Motions of all the heavenly Bodies, and even the Irregularities of the *Moon*, have by this means been ſo nicely obſerved, and by the Power of Numbers reduced within ſome few Tables for common and eaſy Uſe, that their Places for any determinate Inſtant of Time to come are now to be predicted as eaſily, and almoſt as exactly, as we could wiſh.

The nice Prediction Dr. HALLEY gave of the late *Solar Eclipſe*, total at *London*, (a rare Sight in our Part of the *Globe*) is ſuch an Inſtance of the great Perfection to which we are arrived in theſe Matters, as has amazed thoſe unthinking Gentlemen, who were only to be rowzed out of their Security in Ignorance by the Apprehenſions of *Doomſday*. The prodigious Diſtances of the *fixed Stars* beyond that of any of our *Planets*, is, beſides their little or no *annual Parallax*, plainly deducible from the *Teleſcope*,[*] for the longeſt that ever was made, and which perhaps repreſents their *apparent Diameters* one or two hundred times larger than the Truth, has been ſo far from magnifying them, that, by cutting off thoſe irregular Rays which hinder us from diſtinguiſhing the true Termination of their Orbs; it makes them appear ſomething leſſened; beſides that Mr. HUYGENS has given a Method of even computing theſe Diſtances by means of the *Teleſcope*.

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* *Since that Time the Velocity of* Light *has been more exactly determined by the Obſervations of the Reverend and Learned Mr.* BRADLEY, *Savilian Profeſſor of Aſtronomy at* Oxford; *who ſhews, that* Light *comes from the* Sun *to us in* 8 *Minutes and* 13 *Seconds.*

It is now reckoned no abſurd Notion to conceive theſe *fixed Stars* as ſo many *Suns*, probably at as great Diſtances one from another as they are from us, and every one to have their *Syſtem* of inhabited *Planets* circling round it: And perhaps the Number of thoſe which we ſee, counted by HEVELIUS to be 1888, may bear little or no Proportion to thoſe others that may be diſperſed thro' the vaſt Regions of the *Univerſe*, at ſuch Diſtances from our little *Ball*, that no Aſſiſtance can ever help us to a Sight of them. A Notion that gives ſurely the moſt juſt and noble Sentiments that the Mind of Man can entertain of an Almighty Author! That the *milky Way* in the Heavens, which we behold in a clear Star-light Night, is nothing elſe but a continued Cluſter of ſuch *fixed Stars*, is a Truth of which we are aſſured by the *Teleſcope*. And to the ſame Help it is we owe all we know of thoſe heavenly Bodies called *Comets*; their Diſtance, Magnitude, and Motion round the *Sun* in ſuch *eccentrical Orbits*, as come ſome of them very near to right Lines. To what a ſurprizing Height this *cometical Aſtronomy* has been carried by the preſent Age, notwithſtanding the Obſervations we have been able to make upon theſe Bodies are ſo few, and thoſe made by our Predeceſſors ſo imperfect, may be ſeen in the Writings of thoſe incomparable Aſtronomers, Sir ISAAC NEWTON, Dr. GREGORY, and Dr. HALLEY. That in every clear Morning and Evening we ſee the *Sun* for ſome time before he riſes, and after he ſets, is a *Paradox* only to be unriddled by *Dioptrics*: And if we would know the true Place of any heavenly Body elevated not many Degrees above the *Horizon*, the ſame Science tells us, that here *ſeeing* is not *believing*, but that we muſt correct our *Eye-ſight* by a *Table of Refractions*. It is true, the *Ratio* of Refraction of the *Atmoſphere* very near the *Horizon* does not obſerve a conſtant Rule, becauſe there happens a very great Variety in the Accumulation of Vapours about thoſe Parts: But then this Variation depends pretty regularly upon the Poſition of the *Sun* above or below the *Horizon*, and the different State of the Weather; and if in the Morning or Evening we ſee the lower Parts of a diſtant Tower or Mountain through a *Teleſcope* fixed in Poſition, we ſhall find the upper Parts of the ſame Tower or Mountain in the ſame Place, if our Obſervations be made nearer Noon, and juſt at Noon the ſame Object will be ſeen loweſt of all, as the accurate Mr. HUYGENS has obſerved: And this Difference is greater in cold and moiſt, than in hot and dry Weather; and though not in a Proportion always certain, yet conſtant enough for phyſical Purpoſes. The *Crepuſculum* or *Twilight* is determined from the Rays of the Sun below the *Horizon*, firſt refracted at their Entrance into the Earth's Atmoſphere, and then reflected from that Part of it near our *Horizon*, or rather from the contiguous Surface of the *Æther*, as from a *concave Speculum*: And the Height of the *Atmoſphere* has been attempted from this *Theorem* by VARENIUS, but the Air being a *Medium* of different Denſity, and conſequently of different Refraction at different Diſtances from the Earth, refracts the Rays of the *Sun* into *Curves*, and makes that Solution leſs exact. In ſhort, without the Aſſiſtance of *Teleſcopes*, *Aſtronomy* could have come to nothing, and our Obſervations of the Heavens had gone little further than foretelling a fine Morning from the Setting of the *Sun*, or a Shower of Rain from the Courſe of the Clouds. Theſe Inſtances are ſufficient to ſhew that all the noble Diſcoveries of the Heavens, of which the preſent Age may ſo juſtly boaſt of, are derived from the Knowledge of *Catoptrics* and *Dioptrics*; and whatever Improvements are hereafter to be made, can be expected from no other Fountain.

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[*] *The curious Obſervations of the Learned Profeſſor* BRADLEY *abovementioned ſhews, that the* fixed Stars *are much farther from us than was imagined even by thoſe who ſuppoſed that* Light *muſt be ſix Months in coming from them to the Earth: Whereas by Mr.* BRADLEY'*s Obſervations they muſt be* 40,000 *times farther from us than the* Sun; *and conſequently* Light *will be above ſix Years coming from the* fixed Stars *to the Earth. Whence it follows, that if it ſhould pleaſe* GOD *to annihilate one of the* fixed Stars, *we ſhould continue to ſee it ſix Years after it was out. See* Philoſ. Tranſact. *No.* 406.

I ſhall now deſcend to a Proſpect no leſs amazing, which the ſame Science opens to us in the minute Parts of the Creation. The Difficulty which hindered the naked *Eye* from examining the ſmalleſt Particles and ſubtle Texture of thoſe Bodies that are always under our Command, was, that when ſuch *Objects* are brought near enough the

Eye to have their leaſt Parts ſubtend a ſenſible Angle, they become without the Limits of *diſtinct Viſion.* For as long as the *Pupil* of the Eye can, by the circular Fibres of the *Uvea*, be contracted in proportion as the *Object* is brought nearer, the *Cones of Rays* from each *Point* may ſtill be looked upon as *Cylinders*, and will conſequently be brought to a Point in the *Focus* of the *Eye*, which is at the *Retina*, and ſtill make *diſtinct Viſion*: But this Contraction of the *Aperture* of the *Pupil* holding no nearer than about four Inches from the *Eye*, if the *Object* is brought nearer than this, the encreaſed Magnitude is of no further Service, becauſe the *Rays* from each *Point* muſt be now conſidered as *diverging*, and will conſequently after Refraction at the *Eye* be made to converge to a *Focus* beyond the *Retina*, (which is the Place of the *Focus of parallel Rays*) and ſo make *confuſed Viſion*, and the nearer the *Object* approaches, the farther is its *Image* projected beyond the *Focus* of the *Eye*, and becomes ſo much the more confuſed. *Dioptrics* teach us to remedy this Inconveniency two ways, the firſt is, by looking through an *Hole* pricked in a thin *Plate*, ſuppoſe of *Lead*, whoſe *Aperture* muſt be ſo much the ſmaller as the *Object* is nearer, for this ſupplies the Place of a farther Contraction of the *Pupil:* But becauſe this leſſening the *Aperture* excludes a great many *Rays* from each *Point*, and ſo diminiſhes the *Brightneſs* of the *Image*, and that in a *duplicate Ratio* of the *Diameter* of the leſſened *Aperture*, the ſame Science has alſo pointed out to us the more curious Invention of the *Microſcope.* By means of this we diſcern the admirable Range of the *conſtituent Particles* of all ſuch Bodies as come within our nearer View and Acquaintance. The *Cuticula*, or outward Skin of the human Body, is found to be compoſed of ſeveral *Strata* of *Scales*, lying one over another in different Numbers, according to its different Thickneſs in different Places: Between theſe *Scales* the *miliary Glands* diſperſed over the Surface of the whole Body are ſeen to ſend out their *excretory Ducts*, through which we perſpire; and about one of theſe *Scales* the *Microſcope* reckons near 500 ſuch *Ducts*, and that one Grain of Sand will cover 250 ſuch *Scales:* So that one Grain of Sand will cover 125,000 *Orifices* of theſe *excretory Ducts.* A Diſcovery that muſt make us bleſs our ſelves, and ſtand aſtoniſhed at the *Infinity* of the *Creator*, when the Creature is ſo much beyond our Comprehenſion! The inquiſitive Mr. LEWENHOEK has obliged the World with a pro-a prodigious Number of ſuch ſurprizing Truths, which the curious Reader will find among his Writings. The extreme Ductility and Minuteneſs of the Particles of *Gold* is no leſs wonderful; for a Piece of Silver gilt with *Leaf-Gold*, and drawn into the fineſt gilt Wire, whoſe Diameter is $\frac{1}{186}$ of an Inch, and the Thickneſs of the Skin of *Gold* (as Dr. HALLEY has, from the ſpecifick Gravities of the two Metals, computed it) not above $\frac{1}{134500}$ of an Inch, diſcovers not the leaſt Particle of Silver through the Pores of this Skin of Gold, though viewed by the *Microſcope*. The Particles of the Duſt which flies like Smoke out of the *Fungus pulverulentus*, or *Puff-Ball*, when burſt, are diſcerned to be perfect Spherules of an Orange-colour, ſomething tranſparent, and their Diameters not above $\frac{1}{50}$ of that of an Hair: So that a Tube of an Hair's Breadth would contain 125,000 ſuch Spherules. The *Circulation of the Blood*, that noble Diſcovery of our Immortal HARVEY, is now made viſible in the tranſparent Parts of Animals, ſuch as the Fins and Tails of Fiſhes, and the Feet of Frogs; and the *Anaſtomoſes* of the Arteries and Veins put out of Queſtion. It is no leſs inſtructive, and curious to behold the different Organization of the leſſer *Species* of Animals, as the regular

Armour of the *Flea*, the jagged *Proboſcis* of the *Tick*, and the Briſtles of the *Mite*; and in a *Louſe* as he ſtirs his Legs, you ſee the Motion of the *Muſcles* of his Body, whoſe *Tendons* ſeem all to be united in a longiſh dark Spot in the middle of his Breaſt, and the like Motion is obſervable in the *Muſcles* of the ſeveral Articulations of the Legs, and in thoſe of the Head, as he ſtirs his Horns, there alſo appears a great Variety of Branchings of Blood-Veſſels, and the Pulſe regularly beating in ſeveral Arteries, and even the *periſtaltic Motion* of the *Inteſtines*, continued from the Stomach all the way to the *Anus*, which is alſo to be ſeen in the *Flea*, and ſeveral ſorts of tranſparent *Maggots* and *Caterpillers*. Beſides theſe, the *Microſcope* has preſented us with an infinite Variety of little *Animals*, with which the naked Eye can have no Acquaintance. They are obſervable in different Shapes and Sizes about the green Weeds growing in Water, in ſeveral aromatic Infuſions, and in the ſtanding Water in the Hollow of the *Cabbage* and *Teazle*, but in ſuch Numbers in that which drains from an Horſe-Dunghil, that they appear ſometimes as thick as Bees in a Swarm, or Ants on an Hillock, and muſt be diluted with fair Water to ſeparate their different *Species*.

The *Animalcula* in the *Semen virile* are, of all the Subjects, moſt worthy our Notice and Admiration, becauſe from this little ſhapeleſs Creature we have Reaſon to believe that the glorious Frame of Man himſelf ariſes, and this the rather, becauſe in the Seeds of Plants and Trees the *Microſcope* diſcovers the future Plant and Tree already formed, and the *Semen maſculinum* of other Animals, as Bucks, *&c.* are found to be furniſhed with its *Animalcula:* Where it is to be remarked, that ſometimes the Viſcidity of the *Semen* hinders the Succeſs of Obſervations of this kind, and muſt in ſuch Caſes be diluted with a little warm Water. This Theory of Generation is handſomely and at large explained in the *Philoſophical Tranſactions* by Dr. GARDEN. The Uſe of *Microſcopes* has found that loathſome, catching Diſtemper the *Itch* to be occaſioned by the Depredations made upon the Skin by a certain *Species* of voracious *Animalcula*, which are deſcribed in the *Philoſophical Tranſactions* by a Foreigner in a Letter to Dr. MEAD; and indeed ſeems to promiſe the finiſhing Hand to the Science of *Medicine:* For if we can once, by a ſufficient Number of Experiments, determine the different Change of the Texture of the Blood in every different Diſtemper from that which it enjoys in its natural and healthful State, and by mixing the ſmalleſt Particles of ſeveral Medicines with it, find out thoſe which will again reduce it to that natural State, there ſeems to be nothing more wanting to the *practic Part*; and if the true *mechanic Theory* of all theſe different Changes be ever to be known whilſt we live in this Cloud of Fleſh, I'm ſure we muſt have the *Data* for it from the *Microſcope*. The Method of eſtimating the Magnitude of *microſcopical Objects* ſeen by a ſingle *Lens* only, being ſo eaſy that any one who knows ever ſo little of *plane Trigonometry* will eaſily hit of it himſelf, is not mentioned in this Book; beſides that it is already given by Dr. Keill in his *Phyſical Lectures*; where he ſhews that an *Animalculum* placed at the Diſtance of $\frac{1}{10}$ of an Inch before a ſingle *Lens*, and ſeen through it under an Angle of one Minute in Length, is nearly $\frac{3}{100\,000}$ of an Inch long, and if its Figure were cubical, the Magnitude of it would be $\frac{27}{1\,000\,000\,000\,000\,000}$ of a cubick Inch. From whence he concludes with a great deal of Reaſon, that what ſome philoſophical People dream of Angels, may very well be applied to theſe *Animalcula*, that when they have a mind to be merry, ſeveral thouſand Couples of them may lead up a Country-Dance upon the Point of a Needle.

I am ſenſible I need ſay no more in Recommendation of the Subject: And for ſo much of the Book as is Dr. Gregory's, the very Name of the Man gives it ſufficient Reputation. But I am conſcious that Part which I have attempted to add ſtands in need of ſome Name to recommend it with which the World is much better acquainted than with mine: And for that Reaſon I have obtained the Favour of making uſe of thoſe of Dr. Desaguliers and Mr. Jones; Gentlemen againſt whoſe Judgment in theſe Matters their Approbation of the following Papers is the only poſſible Objection; and whoſe Names can never fail of meeting with that Eſteem which they deſerve, when fixed to any thing of their own, however they may happen to be treated for appearing in this Place to recommend what is mine.

W. Browne.

THE ELEMENTS OF CATOPTRICS.

PROP. I. THEOR. I.

IF a Ray of Light be reflected from a plane Surface, the Angle of Reflection will be equal to the Angle of Incidence: [Plate 1. Fig 1.]

Tho' the Demonſtration of this Theorem belongs more propetly to Phyſics, and a Mathematician might very well take it for granted, as ſufficiently proved by Experiments; yet in Compliance to Cuſtom, we ſhall here give a Demonſtration of it, and That ſuch a one, as ſhall have no Dependance upon any Party of Philoſophers whatever.

Let AB ſignify a Ray proceeding from the Radiant Point A, and falling upon the plane reflecting Surface, GC at B. I ſay its reflected Ray BE will be ſuch, that the Angle of Reflection PBE (BP being ſuppoſed perpendicular to GC) will be equal to the Angle of Incidence PBA.

From the Radiant Point A, let fall AC perpendicular to the Surface GC. All the Rays comprehended between AB and AC, and which poſſeſs, in the reflecting Plane, the Right Line BC (the common Section of the Plane of Inflection with the reflecting Plane) would, if the Medium GC *de* were penetrable to *Light* and homogeneous with the former EBCD, be diffuſed in the ſame conſtant Tenour; that is, at the Diſtance B*e* beyond GC, they would be diffuſed into *de*, AB*e* and AC*d* being ſuppoſed Right Lines. And ſince, by Suppoſition, the plane reflecting Surface BC, neither encreaſes nor diminiſhes the Diffuſion of thoſe Rays (for if it were otherwiſe, which no Experiments yet give us any reaſon to ſuſpect; it is plain the Angle of Reflection as well as of Refraction would, for the ſame Reaſons, become unequal to the Angle of Incidence: For the due inflecting of which Rays, CARTESIUS, has contrived ſuch Conoids as are proper, (*Geom. lib.* 2) but only ſtrikes them back upon the ſame Medium in which they were before;

fore; that is, upon a Medium where the Rays are diſperſed or diffuſed after the ſame manner as at firſt; it is plain, that after they are reflected from the Surface G C, the Rays will be diſperſed after the ſame manner, as they would have been, if they had never met with the Surface C G, but, (that being removed,) had ſtill remained in the ſame Medium. In which caſe, when the Diſtance is increaſed by the Exceſs B *e*, the Rays would be diſperſed thro' the Space *ed*. Wherefore it is plain, that at the Diſtance B E equal to B *e*, they will be diſpers'd thro' the Right Line D E, equal to the Right Line *de*. But the Ray A C is reflected upon it ſelf, wherefore D E is equal to *de*, and the Figure B C D E every way ſimilar and equal to the Figure B C *de*; and indeed is the very ſame reflected, or revolved half way about the fixed Right Line B C. Therefore if from the equal Angles E B C, *e* B C, the right Angles P B C, A B C be taken away, the Angles E B P, *e* B *p* will remain equal. But A B P is equal to *e* B *p*, therefore the Angle of Incidence P B A, is equal to the Angle of Reflection P B E. *Q. E. D.*

SCHOLIUM.

In laying down this Law of Reflection, whereby the Angles A B P, E B P, are made equal, great Reſpect is had to the *Maximum* and *Minimum*. For the Sum of the Right Lines A B, B E is a *Minimum*; that is, leſs than the Sum of any others drawn from the ſame Points A and E, to any other Point of the Plane G C: and ſo *vice verſa*. For it is requiſite that the Courſe which the Ray (reflected from the Plane G C) takes from A to E, ſhould be the ſhorteſt of all. Becauſe *Nature ever acts by the moſt eaſy and expeditious Methods*. But if the Reflection be from a curve Surface, the aforementioned Sum of Rays (or perhaps their Difference) is ſometimes a *Maximum*. For Mathematicians know how near the Relation is between a *Maximum* and a *Minimum*, between the Sum and the Difference, and how eaſy the Tranſition is from one to the other.

COROL. I.

Hence the Angles E B G, A B C (which are likewiſe by ſome call'd the Angles of Incidence and Reflection) are alſo equal: for they are the Complements of the former to right Angles.

Corol. 2.

If the Reflected Ray be looked upon as an Incident Ray, the Incident Ray will *vice verſa*, be its Reflected Ray.

Corol. 3.

If the Angles P B E, P B A, or the Angles G B E, C B A be equal, B E will be the Reflected Ray belonging to A D.

Prop. II. Probl. I.

The Focus *of Rays falling upon a plane* Speculum *being given, to find the* Focus *of thoſe Rays after their Reflection from the Surface of the* Speculum. [Plate 1. Fig 2.]

From the given *Focus* A, draw the Right Line A C perpendicular to the plane *Speculum* C F, which produce to *a*, that C *a* may be equal to C A; *a* will be the *Focus* required. Let A D be an incident Ray, join *a* D, and produce it.

Becauſe in the Triangles A C D, *a* C D the Sides A C, *a* C, are equal, and the Angles A C D, *a* C D alſo equal, becauſe both right, and the Side C D is common to both, therefore the Angle A D C is equal to *a* D C; but *a* D C is equal to E D F; therefore A D C, E D F

EDF are equal. And conſequently DE, by *Corol.* 3. *Prop.* I. is the Ray reflected from the plane *Speculum* CF, belonging to AD. The ſame may be ſhown of any other Ray proceeding from the Point A, and falling upon the *Speculum*. From whence it is plain, that the Rays diverging from A, after their Reflection from the plane *Speculum* CF, will diverge from the Point *a*: that is, thoſe Rays, whoſe *Focus* before Reflection was A, will after their Reflection have *a* for their *Focus*. *Q. E. D.*

SCHOLIUM.

Since the Eye in any Poſition, as at O, will receive the Rays thus reflected, after the ſame manner, as if they really had proceeded from the Point *a*, it is plain the Image muſt appear in that Place: Becauſe the Rays diverging from the Point *a*, affect the Eye after the ſame manner, as if that were the primary Radiant.

COROL. I.

From this Prop. and *Corol.* 2. *Prop.* I it follows, that the Rays ED, OB converging towards the *Focus* *a*, will after their Reflection from the *Speculum* CF, converge towards the *Focus* A: and that ſuch a Poſition of the Plane CF, may be eaſily aſſigned, as ſhall make the Angle O B A equal to any given Angle; which is done by taking O B F equal to half the Complement of the given Angle to two right ones.

COROL. 2.

In a plane *Speculum*, the Image of any radiant Point is ſeen in that Place where the Reflected Ray O B which paſſes thro' the Centre of the Eye, meets with the perpendicular A C, let fall from the radiant Point upon the *Speculum*. Whence, tho' every part of the *Speculum*, except B, were covered or taken away, the Image would nevertheleſs be viſible: and if the *Speculum* at B be covered, tho' all the reſt be open, the Image will not be ſeen at all.

COROL. 3.

The Images of Objects that are inclined to the Plane of the *Speculum*, are inclined to the plane of the *Speculum* after the ſame manner as the Objects themſelves; and therefore in a plane Horizontal *Speculum*, vertical Objects appear inverted.

This and what follows will be eaſily underſtood, if the Object be conceived as made up of ſeveral radiant Points; and the Image of every one be attended to: For of all theſe together the Image of the Object conſiſts.

COROL.

Corol. 4.

A radiant Plane and its Image made by a plane *Speculum*, are equal and ſimilar Figures, but not ſimiiarly placed. They differ as the Right Hand from the Left, or as a Figure engraven upon a Copper Plate, does from the Impreſſion of it taken upon Paper. For they will fit if they come together.

Corol. 5.

Becauſe the Right Line *a*B is equal to AB, *a*O will be equal to AB and BO together; that is, the Diſtance of the Image from the Eye, is equal to the Incident and reflected Ray taken both together.

Corol. 6.

Whatſoever has been ſaid of the Image of any Object or primary Radiant, holds true alſo of the Image of another Image. From whence ariſes that Multiplication of Images, made by two or more plane *Specula* duly poſited; in which it is principally to be obſerved, that the Diſtance of any Image from the Eye, is equal to a Ray propagated from the primary Radiant, thro' all the intermediate Reflections to the Eye.

LEMMA.

A Ray of Light is inflected by a curve-Surface after the ſame Manner, as it wou'd be by a plane Surface, touching the Curve in the Point of Incidence. *Fig.* 3.

Let P Q be any curve Surface, (for here, in the Figure, as likewiſe in all Caſes hereafter, the common Sections of the inflecting Surface, and the touching Plane, with the Plane of Inflection, are uſed for the inflecting Surface, and the touching Plane themſelves: becauſe every Inflection is perform'd in the Plane of Inflection, as is demonſtrated in *Axiom* 2. And for the ſame Reaſon in the Room of a ſolid Figure, we make uſe of a plane one deſcribed in the Plane of Inflection; which ſaves us a great many Words) upon which the Ray A B falls at B. Now ſince the Ray is of a Thickneſs not conſiderable, the Particle, in the Curve Surface B, upon which it falls, will be extremely ſmall. But the Inclination or Direction of the Curve Surface at B, is the ſame with the Inclination of the Plane D E touching it in that Point: Wherefore the Inflection which depends upon the Direction, is likewiſe the ſame, whether it be conceiv'd, as occaſion'd by the Particle B in the Curve Surface P B Q, or in the plane one D B E. For as to the Inflection of the Ray A B, it matters not how the reſt of the Surface is bent, if the Inclination of the ſmall Particle B, upon which it falls, does but remain the ſame.

It wou'd be eaſy to demonſtrate the ſame Thing, as the Antients did, from hence, that the Angle of Contact D B P, or E B Q, is leſs than any rectilineal one; and that a Plane may be found ſo inclined to the Plane D B E, that the Difference between the Inflections that are made by them both, ſhall be leſs than any given Inflection. For from hence it will follow, that the Inflection of the Ray A B, made at the Particle B of the Curve Surface, is no way different from that which is made at the Particle B of the plane Surface D B E, touching the Curve Surface in B.

PROP. III. PROB. II.

TO find the Focus *of Parallel Rays falling upon a given Spherical* Speculum, (*or to find the Image of a vaſtly diſtant radiant Point*) *with reſpect to an Eye placed in the* Axis *of the* Speculum, *which is parallel to the incident Rays.*

Thro' A the Centre of the *Speculum*, draw the right Line A B parallel to the incident Rays, meeting the *Speculum* in B. Biſſect A B in C, I ſay C is the *Focus* required.

In this, as well as in all the following Propoſitions, we ſuppoſe the Point D to be extremely near the Point B. And this is neceſſary, in order that the reflected Ray belonging to the incident one E D, may meet with the Eye, which, by Suppoſition, is placed in A B, or the ſame produced: For the reflected Rays belonging to thoſe that fall more remote from the Point B, do, after their diverging from their *Focus*, paſs beyond the Pupil of the Eye; and conſequently contribute nothing towards ſeeing the Image. Beſides, of thoſe Rays that enter the Pupil of the Eye, they that fall moſt directly, or neareſt the Middle of the Pupil; (that is, that are reflected from thoſe Points that are neareſt to the Point B,) conduce more towards ſeeing the Image, than thoſe that enter the Eye near the Extremes of the Pupil: Becauſe thoſe that fall moſt directly, and cloſe to one another, move the Senſe more forcibly, than thoſe that fall more oblique and ſcatter'd. For which Reaſons, we need have reſpect only to thoſe that fall neareſt to the *Vertex* B, which holds good in all the Propoſitions following.

Fig. 4. Let E D be one of the parallel incident Rays. Draw A D, C D, and produce them. Becauſe the Point D almoſt coincides with B, C D will be nearly equal to C B: But by Conſtruction, C A is equal to C B; therefore C A, C D are equal; and therefore the Angle C A D, C D A are likewiſe equal. But the Angle C A D is equal to E D A, which is the Angle of Incidence; for the right Line A D is perpendicular to the Surface of the Sphere. Whence the Angle C D A is equal to the Angle of Incidence of the Ray E D. Therefore, by *Corol.* 3. *Theor.* 1. D C is the reflected Ray belonging to the incident one E D. Moreover. the Angles at the *Vertex*, E D A, *e* D *o*, and C D A, N D *o* being equal, the reflected Ray N D, belonging to the Ray *e* D parallel to B A, and falling upon the convex Surface, will, if produced backward, go to C. And what is demonſtrated of any one Ray E D taken at pleaſure, is true of all the reſt in the ſame Circumſtances. Wherefore, ſuch Rays as are parallel to A B, and which, when reflected, conduce to Viſion, if they fall upon a Concave Sphere, are, after Reflection, collected in C: and from thence again diverging, make the image to be ſeen in that Place, by an Eye placed in the *Axis*. The reflected Ray belonging

to the very *Axis* it ſelf does alſo, as it were, diverge from the Point C, in the Middle between B and A: For the ſame Thing happens to it, as to any other Ray reflected from B D, that cuts the right Line A B in a Point equally diſtant from the Centre A, and the Point of Incidence. But Rays that fall upon the Convex Side, do after Reflection, diverge from C, and make the Image appear in the Point C, (or which is all one, in a Point whoſe Diſtance from C is leſs than any given one) to an Eye placed in A B produced. *Q. E. D.*

COROL.

Hence, and from *Corol.* 2. *Theor.* I. it follows, that Rays diverging from C, and reflected from the Concave Surface, or converging towards C, and reflected from the Convex Surface, will be parallel to the right Line A B, joining the Centre of the Sphere, and the Point C.

SCHOLIUM. *Fig.* 5.

If with the Vertex B, the *Axis* B G, and a Parameter equal to the right Line F B, a Parabola be deſcribed, it will be the *leaſt* of all that can be circumſcribed about the given Circle F B; or the Circle F B will be the *greateſt* of all that can be inſcribed within that Parabola, by *Corol.* 1. *Prop.* XX. *lib.* 1. *Vincentii Viviani de Maximis & Minimis.* And this Circle and Parabola will, at the Point B, *have the ſame Degree of Curvity,* (to uſe KEEPLER's Words, in *Cap.* VII. *Prop.* XX. *Paralipom. in Vitellionem*) and are there moſt intimately united. For as the Contact of Lines is equivalent to two Interſections, and is really no way different from two Interſections infinitely near to one another, as Mathematicians know very well, and for the ſame Reaſon Surfaces that mutually touch one another, have the ſame Power in inflecting Rays that fall upon the Place of Contact, and in producing other Phyſical Effects, as is ſhewn in the foregoing *Lemma* : So this more intimate Union, and which is equivalent to four Interſections, and conſequently the moſt intimate that can be between the Circle and conick Section, (as the Contact between this laſt and a right Line is of all the moſt intimate) this, I ſay, will, in Phyſical Effects, that depend upon Surfaces generated by the Revolutions of theſe Lines, produce a farther Equipollency. For as Rays that are parallel to G B, are by the concave Parabolick Conoid B D, reflected exactly to its *Focus* C, which is diſtant from the Vertex B, by a fourth Part of the Parameter of the generating Parabola ; ſo the ſame Rays reflected from the concave Sphere, are collected very nearly in the ſame Point; and the ſame Speculation holds in ſeveral other Phyſical Matters.

PROP. IV. PROBL. III.

THE Focus *of Diverging Rays being given, whoſe Diſtance from a given concave Spherical* Speculum, *is greater than the fourth part of its Diameter; To find the* Focus *of theſe ſame Rays after their Reflection from the forementioned* Speculum, *with reſpect to an Eye placed in its* Axis. Fig. 6.

Through A the Centre of the Sphere, and the given *Focus* E, draw a right Line, meeting the Spherical Surface in the *Vertex* B; this I call *the Axis of Radiation*; and ſuppoſe the Eye to be placed in it ſome where or other. In this right Line take the Point C ſuch, that B C may be to C A, as B E is to E A. I ſay C is the *Focus* required.

Let any Ray proceeding from E, fall upon the Concave Surface of the Sphere at the Point D, near enough the *Vertex* B, (for we have nothing to do with thoſe that fall more remote, becauſe after Reflection they affect not the Eye placed in E B, as has before been ſhown) in which Caſe E D will be nearly equal to E B, and may in

Phyſical Matters be taken for it. Draw A D, C D. After the ſame manner C D is equal to C B; and from whence by Conſtruction C D will be to C A, as D E to E A. And inverting the Proportion, C D is to D E, as C A to E A. Wherefore by *Euc. Elem.* VI. 3. the Angles A D E, A D C are equal; but A D E is the Angle of Incidence of the Ray E D, and conſequently D C (by *Corol.* 3. *Theor.* I.) is the Ray E D reflected from the concave *Speculum.* And ſince the Ray E D is taken at Pleaſure, it is plain the *Focus* of all the Rays diverging from E, after Reflection, will be in C, with Reſpect to an Eye placed in the *Axis* E B. *Q. E. D.*

COROL.

Hence likewiſe will be given the *Focus* C, of Rays *e* D converging, in the forementioned Circumſtances, towards the given *Focus* E, and reflected from a given convex Spherical *Speculum.*

SCHOLIUM. *Fig.* 7.

Rays diverging from E, are reflected from the Concave Sphere B D converging to C, for this reaſon; becauſe the Circle B D deſcribed on the Centre A (by whoſe Rotation the Sphere is generated) has the ſame Degree of Curvity with an *Ellipſis*, deſcribed upon any *Foci* C and E (found as in the *foregoing Prop.*) through B, and generating an oblong Spheroid, by which, as is commonly known, Rays diverging from one of the *Foci*, are from the concave Surface reflected, converging towards the other of them.

Draw the leſſer *Semi-axis* H K, join the right Lines C K, E K, each of which are equal to the *Semi-axis* B H. From the Centre H upon C K, let fall the perpendicular H L.

It is plain L K is half the *Latus Rectum* belonging to the *Axis* B G, becauſe it is a Third proportional to the right Lines C K and H K. (Which is alſo true, if from H the Point of Interſection of the greater *Axis*, with the right Line K H biſecting the Angle comprehended by the right Lines drawn from the *Foci* to any Point of the Curve, a right Line be let fall perpendicular upon either of the foreſaid right Lines.) And conſequently C L is half the difference between the greater *Axis* and its *Latus Rectum*; and is alſo a Third proportional to the right Lines, K C, C H, that is, to the greater *Semi-axis*, and half the diſtance of the *Foci*. Again, (by the Conſtruction in the *preceeding Prop.*) B C is to C A, as B E to E A; from whence by changing and inverting the Proportion, B E will be to B C (E G) as A E to A C. Therefore B E + E G. B E — B C : : A E + A C.
D A

AE—AC, that is, BG is to CE, as CE to AE—AC. And taking the halves of them, BH is to HC, as HC to HA: That is, HA is a Third proportional to the greater *Semi-axis*, and half the Diſtance of the *Foci*; whence it is equal to half the difference between the greater *Axis* and its *Latus Rectum.* Wherefore AB is half the *Latus Rectum.* And conſequently the Circle BD is (*by Corol.* 1. *Prop.* XX. *lib.* 1. *Vinc. Viviani*) the greateſt of all that can touch the Ellipſis BG on the inſide at B. Wherefore (as is ſhown in the *Scholium* of the *preceeding Prop.*) it will in B have the ſame Curvity with the forementioned Ellipſis. From whence it comes to paſs, that a Sphere thence generated, performs the ſame Thing in reflecting very nearly, which a Spheroid generated by the Ellipſis does exactly.

PROP. V. PROBL. IV.

THE Focus *of diverging Rays being given, whoſe Diſtance from a given Concave* Speculum, *is leſs than the Fourth Part of the Diameter of the* Speculum; *To find their* Focus *after Reflection from the forementioned* Speculum, *with reſpect to an Eye placed in the* Axis. *Fig.* 8.

Through A, the Centre of the Sphere, and the given *Focus* E, draw a right Line, meeting the Spherical Surface in B. In this take the Point C ſuch, that B C may be to C A, as B E to E A. I ſay C is the *Focus* required.

Let E D be any one of the incident Rays, draw A D, C D, and produce them; draw likewiſe the right Line, E R, parallel to the right Line C D.

Since the Arch B D is extremely ſmall, E D, E B, and C D, C B will be equal. Wherefore E D is to E A, as C D to C A; that is, becauſe (D C, E R are parallel) as E R to E A; therefore the right Lines, E R, E D, and conſequently the Angles, E R D, E D R are equal. But E D R is the Angle of Incidence of the Ray E D, and E R D is equal to its alternate N D A, wherefore (by *Corol.* 3. *Theor.* 1.) D N is the reflected Ray belonging to the incident one E D. And ſince E D is taken at pleaſure, it is plain that all the Rays proceeding from E, after they are reflected from the Concave Spherical Surface, will, if they be produced backwards, meet in C, or will have their *Focus* in C. *Q. E. D.*

Corol.

Hence, and from *Corol.* 2. *Theor.* 1. may be found the *Focus* E, of the Rays N D, converging towards a given *Focus* C, and reflected from a given concave ſpherical Surface B D.

Prop. VI. Prob. V.

T*HE* Focus *of Diverging Rays being given; To find their* Focus *after Reflection from a given Convex ſpherical* Speculum, *with reſpect to an Eye placed in the* Axis. *Fig.* 9.

Thro' A, the Centre of the Sphere, and the given *Focus* E, draw a right Line meeting the ſpherical Surface in B: In this take the Point C in ſuch manner, that A C may be to C B, as A E to E B. I ſay C is the *Focus* required.

Let E D be any incident Ray proceeding from E, draw C D and produce it. To this thro' E, draw E R parallel, meeting the right Line A D, produced in R. The Arch B D being evaneſcent, for the Reaſons already given, C D will be equal to C B, and E D to E B; and therefore A C is to C D, as A E to E D, but becauſe of the equi-angular Triangles A C D, A E R, A C is to C D, as A E to E R. Therefore A E is to E D, as A E to E R. Wherefore E R is equal to E D. Therefore the Angle E R D, or its equal N D R, is equal to the Angle E D R, that is, to the Angle of Incidence of the Ray E D. From whence, it follows, that D N is the reflected Ray of the incident E D. And ſince E D is taken any how, it is plain, all the Rays diverging from the *Focus* E, and entring the Eye, whoſe Poſition is given, will, after their Reflection from the convex ſpherical Surface B D, diverge from the *Focus* C. *Q. E. D.*

COROL.

Hence may be found the *Focus* E, towards which the Rays N D (converging before Reflection towards a given *Focus* C, whoſe Diſtance from the *Speculum* is leſs than the Fourth Part of its Diameter,) do, after their Reflection from a given convex *Speculum*, converge.

SCHOLIUM. *Fig.* 10.

It had been eaſy to have explained and demonſtrated theſe two laſt Propoſitions, and their Corollaries, in the ſame Words, and with different Figures, but this would have bred Confuſion to Beginners. However, one *Scholium* will ſerve for both, to ſhow that the Circle B D has the ſame Degree of Curvity, with the Hyperbola deſcribed upon the *Foci* C and E, through the Point B, by the Rotation of which Curve round its *Axis*, is generated the Surface of an Hyperbolick Conoid, performing exactly the propoſed Reflection.

Draw the ſecond Diameter K M. To the joined right Line B K, erect K L perpendicular, meeting the *Axis* G B in L, and H L will be a Third proportional to the right Lines, B H and H K, and conſequently equal to half the *latus Rectum* belonging to the *Axis* B G. And ſince, from the Nature of the *Foci*, the Rectangle G C B is equal to the Square of the right Line K H; adding to both the Square of B H; the Squares of the right Lines B K and H C will be equal: Wherefore B K, H C are equal. And becauſe of the Rectangular Triangle B K L, B L, or the Sum of half the Tranſverſe *Axis*, and half the *latus Rectum*, is a third proportional to the right Lines H B, B K; or to the right Lines H B, H C; That is, to half the Tranſverſe *Axis*, and half the Diſtance of the *Foci*. Again (by the Conſtruction in the *two preceeding Prop.*) B C is to C A, as B E to E A. Wherefore B E is to E G, as A E to A C. Therefore B E—E G. B E + E G :: A E — A C. A E + A C, that is, B G is to C E, as C E to A E + A C. And taking the halves of them, B H is to C H, as C H is to H A: That is, H A is a third proportional to half the Tranſverſe *Axis*, and half the Diſtance of the *Foci*. Therefore H A is equal to B L, or to the Sum of half the Tranſverſe *Axis*, and half the *latus Rectum*. Wherefore A B is half the *latus Rectum*, and conſequently the Circle B D is (by *Corol.* 1. *Prop.* 20. *lib.* 1. *Vinc. Viviani*) the greateſt of all that can touch the Hyperbola B G on the inſide at B. Therefore, as has been ſhown already, it is equally curve in B with the Hyperbola. And hence it is, that the Surface of the Sphere, generated by the Circle, performs *very nearly* the ſame Thing in reflecting, which the Surface of the Hyperbolick Conoid does exactly: That is, it changes the *Focus* of the Rays E, into C; or the *Focus* C into E, as has been ſhown in *Prop.* V. and VI.

PROP.

Prop. VII. Prob. VI.

T*HE* Focus *of Rays falling upon a given ſpherical* Speculum *being given; To find the* Focus *of the ſame Rays after Reflection, with reſpect to an Eye (even any where out of the* Axis) *given in Poſition.* Fig. 2.

Hitherto we have ſuppoſed the Eye placed in the *Axis* of Radiation; that is, in a right Line drawn through the radiant Point, and the Center of the reflecting Sphere, both becauſe moſt optical Inſtruments are made after this Manner, and becauſe the Image ſeen by the Eye ſo placed, is much more lively and diſtinct than any other, becauſe it is formed by Rays leaſt ſcattered, and moſt exactly reflected, and upon this Account challenges to it ſelf alone the Title of an Image.

But that it may appear that the Method before uſed in conſtructing Phyſical Problems, and demonſtrating them when conſtructed, is more univerſal, and ſufficient to determine the Image, ſeen by the Eye, however placed: Let the reflecting Sphere be ſignified by its greateſt Circle BD, whoſe Plane paſſes through the radiant Point E, and O the Center of the Eye. It is required to find the Image of the Point E made by made by the reflected Rays, with reſpect to an Eye placed at O.

It is plain in the firſt Place, that the Image will be found ſomewhere in the reflected Ray paſſing thro' O. To find that Ray, this Problem muſt be ſolved, by which having two Points E and O, and (in the ſame Plane) the reflecting Circle given; it is required to find ſuch a Point in the Circumference of the Circle, that a Ray falling upon it from either of the given Points ſhall be reflected to the other of them. And this is *Prop.* XXXIX. *Lib.* V. *Opticæ Alhazeni*, to which he has premiſed 7 or 8 *Lemmata*, and is now commonly called *Alhazen's Problem.* The Problem is in its Nature *ſolid*, and not to be conſtructed univerſally without the Interſection of a Conick Section with the given Circle. The Conſtruction of this Problem has been publiſhed by ſeveral eminent Geometers, *Barrow*, *Sluſius*, &c. but moſt elegantly by the moſt *noble C. Huygens* in the *Philoſophical Tranſactions*, No. 98. We proceed therefore to determine exactly the Point it ſelf C, where the Image is ſeen in the right Line O B, (drawn as thoſe famous Men have directed).

Produce E B, O B, till they again meet the Circle in P and R. Biſect the Right Lines B R, B P in S and A, and divide S B in C ſo, that S C may be to C B, as

A E to E B. I ſay the Image by the Eye in O will be ſeen in C; and not in the Meeting of the reflected Ray D N with the right Line E Q, joining the radiant Point and the Centre of the Sphere, as *Euclid* in *Theor*. 17. and 18. *Catoptrica*, and others would have it.

Let the Ray E D fall upon the Point D, very near the Point B: For only thoſe Rays, that fall after this Manner, do, after their Reflection, enter the Pupil of the Eye placed at O. Thoſe that fall at a greater Diſtance, after Reflection, paſs beſide the Eye, and conduce nothing at all to Viſion. Wherefore in inveſtigating the Point C, where the Image is formed, D muſt be ſuppoſed to coincide with B: In which Caſe $ED = EB$, $CD = CB$, $AD = AB$, and $SD = SB$. Moreover, becauſe O B is the reflected Ray of E B, the Angles A B Q, S B Q will be equal; and the Point D approaching near to B, and at laſt coinciding with it, the Angles A D Q, S D Q coinciding with the former A B Q, and S B Q (at leaſt being very little different from them) will in that Caſe be likewiſe equal. But farther, the Point D approaching to B, the ultimate Angles, or the ſmall and evaneſcent ones, D A B, D Q B, D S B will alſo be equal: For the Circle drawn through Q, and the coinciding Points D and B; that is, deſcribed on the Diameter B Q, and conſequently touching the Circle B P Q R on the inſide, paſſes through the middle Points of the right Lines B P, B R, howſoever drawn from the Point B: And conſequently the Angles D A B, D Q B, D S B, in this Caſe, are in the ſame Segment of the Circle paſſing through the Points D, B, S, Q and A. Theſe Things being laid down, which follow from the Coincidence of the Point D with B; ſince, by Conſtruction, A E is to E B, as S C to C B, A B will be to D E, as S C to C D. And ſince in Triangles, the Sides are as the Sines of the Angles ſubtended by them; the Sine of the Angle A D O, or M D A, will be to the Sine of the Angle D A B, as the Sine of the Angle C D S is to the Sine of the Angle D S B; becauſe therefore the Conſequents are equal, the Antecedents will likewiſe be equal; and conſequently the Angles to which they relate, namely A D M and C D S, are equal. But it has been ſhewn before, that the Angles A D Q, S D Q are equal: Therefore the Angles Q D M, Q D C, and thoſe oppoſed to them at the *Vertex* L D E, L D N are equal. Wherefore N D is the reflected Ray of the Incident one E D. And the ſame may be ſhewn of the other Rays, that meet with the Eye at O; namely, that their reflected Rays will diverge from the Point C. Wherefore to the Eye receiving theſe Rays only, the Image of the radiant Point E, made by Reflection, will appear at C. *Q. E. D.*

Corol. 1.

Hence it follows, that Rays converging towards the Point C, will, after their Reflection from a convex ſpherical *Speculum*, converge towards E, and there form the Image, for an Eye placed in the right Line B E, any where beyond E. As likewiſe that the Image of the radiant Point C, made by Reflection from a concave ſpherical *Speculum*, will, to an Eye placed in the right Line E B produced, as in A, appear in E.

Corol. 2.

But if the radiant Point E be vaſtly diſtant, then the *Ratio* of the right Line A E to E B becomes a *Ratio* of Equality, and by Conſtruction S C is equal to C B: That is, the Image of the vaſtly diſtant radiant Point ſituated in A B produced, made by a convex or concave *Speculum*, with reſpect to an Eye placed any where in B R, or the ſame produced, will be in the middle Point of the right Line B S.

The Eye being placed in the *Axis* of Radiation, that is, the right Lines B A and

B S coinciding with the right Line B Q, this ſame Conſtruction ſerves, and degenerates into the Conſtructions of the four preceding Propoſitions.

But the ſpherical Surface being changed into a plane one, then this Conſtruction will be changed into that of *Prob.* I. For becauſe the right Lines A E, S C are in that Caſe infinite, but their Difference remains ſtill finite, the *Ratio* of them will be a *Ratio* of Equality: Therefore E B and B C, which are proportional to them, are equal. From whence it is plain, that this one Conſtruction of this *Prob.* VI. contains all the former ones, as being more ſimple; which is frequent in Geometry.

SCHOLIUM.

The fore-mentioned Properties do therefore belong to the Circle B D R, becauſe it has the ſame Curvity at the Point B, with a conick Section deſcribed through B upon the *Foci* C and E; which, as is known, reflects the Rays diverging exactly from C. The Equality of Curvity in the foreſaid Caſe is from thence manifeſt: That the Segment of the Diameter of the Section (produced if need be) cut by the equicurve Circle, is equal to the *Latus Rectum* of that Diameter: And *vice verſa.*

PROP.

PROP. VIII. THEOR. II.

THE *Image of a radiant Plane Surface made by a Spherical* Speculum, *is alſo a Plane Surface.* Fig. 12, 13.

Hitherto we have determined the Image of any radiant Point with reſpect to an Eye, placed, either in the ſame *Axis* of the *Speculum* with the radiant Point, or out of it. From whence it is eaſy to determine the Images of radiant Bodies, becauſe made up of radiant Points. But to the more eaſy Determination of it, as likewiſe to many other Things belonging to Practice, the *Theorem* before us will not a little conduce: In which we make uſe of that Image of any Point, which is ſeen by an Eye placed in the ſame *Axis* with the radiant Point. For we ſpeak not here of thoſe Images, that are ſeen by the Eye in any other Poſition (which yet may be determined by the Help of the preceding *Prop.*) becauſe they are only ſecondary and leſs conſiderable; eſpecially ſince ſuch an Image of a right Line has been already ſufficiently conſidered by the *famous Barrow*, in *Lect. Opt.* XVI. and XVII.

Let any ſpherical *Speculum* be ſignified by its greateſt Circle B D, whoſe Centre is A; and the radiant Plane by the right Line F E. From A upon F E, ſuppoſe A E drawn perpendicular, meeting the *Speculum* in the *Vertex* B. Find, by ſome one of the preceding *Prop.* applicable to the Caſe, the *Focus* C of thoſe Rays after Reflection, whoſe *Focus* before Incidence was E; through this draw the Plane C T parallel to F E. I ſay that the Image of the Plane F E made by the *Speculum*, will be placed in the Plane C T.

From any Point F of the radiant Plane, to A the Center of the *Speculum*, ſuppoſe the Ray F A produced, meeting the *Speculum* in D, and the Plane C T in T. The reflected Ray of the incident one E D, is, by Conſtruction, C D; but let B H be ſuppoſed the reflected Ray of F B, meeting the Ray F D in H.

Becauſe the Angle F A E is, by Suppoſition, ſmall, the Arch D B will be alſo ſmall, and almoſt degenerate into a ſmall right Line. And the Circumference of a Circle deſcribed on the Diameter B F, will paſs through the Points D and E; becauſe of the right Angles F E B, F D B. Whence the Angles B F D, B E D, in the ſame Segment, are equal. And ſince the Angles oppoſed at the *Vertex* A are equal, or the ſame, F B A, E D A will be likewiſe equal: Therefore the Angles A B H, A D C, that are by *Prop.* I. equal to theſe, are likewiſe equal. Whence the Triangles A B H, A D C are equiangular, and conſequently ſimilar. Wherefore A B is to A H, as A D to A C; and ſince A B is equal to A D, A H will be equal to A C. But becauſe of the Smallneſs of the Angle C A T, A T is reckoned equal to A C; therefore A H is to be reckoned equal to A T. But H is the Image of the Point F in the radiant Plane; whence its Poſition is in the Plane C T. The ſame may after the like manner be ſhewn of any other Point in the Plane F E: From whence the Propoſition is manifeſt.

But if the Object expoſed to the *Speculum* be vaſtly diſtant, the Images of each of the radiant Points will, by *Prop.* III. be in the middle Points of the Semidiameters of the *Speculum* tending towards them: That is, the Image of a diſtant Object will form a ſpherical Surface, concentrical with the *Speculum*. But becauſe the radiant Body, by Suppoſition, is ſeen under a very ſmall Angle, ſo ſmall a Portion of the ſpherical Surface as is poſſeſſed by its Image, ſcarce differs from the plane Surface to which A B is perpendicular. Wherefore the Propoſition holds in all Caſes: For the Demonſtration takes Place in any other Caſe, as well as in thoſe two expreſſed in the Figures.

COROL.

If the Angle E A F be too great, the right Line A H will be ſenſibly leſs than A T: whence the Image of an Object ſeen under too large an Angle, from the Center of the *Speculum*, made by a concave *Speculum*, will be ſenſibly concave; and that which is made by a convex *Speculum* will appear convex.

SCHOLIUM.

Concerning the Images of radiant Surfaces, made by ſpherical Surfaces, it will not be improper to take notice of ſome few Things. 1. If the Object expoſed to the concave *Speculum* be diſtant from it more than by its Semidiameter, the Image of it, C T, will be diſtant from the *Speculum* more than by a fourth part of its Diameter, but leſs than by its Semidiameter; that is, it will appear hanging in the Air between the Object and *Speculum*, and likewiſe inverted; that is, the upper Parts will appear undermoſt, and the right on the left. If the Object recedes from the *Speculum*, the Image will approach to the *Speculum*; but if the Object approaches to the *Speculum*, the Image will recede from it, till meeting at laſt with the Object, it will coincide with it in the Center. After the ſame manner the Image of the Radiant C T will be F E, whoſe Properties are evident by what has been juſt now ſaid. But if the Diſtance B E be ſo far encreaſed that the Object be vaſtly diſtant, the Diſtance of the Image C T from the *Speculum* will be equal to a fourth part of its Diameter. But if the Sun be the radiant Body, in the room of its Image will be excited a Burning (if the *Speculum* be conſiderably larger than the Image of the Sun) becauſe of the Sun's Rays being cloſer compacted in that Place. But if a lucid Body be placed in the Middle, betwixt the *Vertex* and Center of the *Speculum*, its Image made by the *Speculum* at a vaſt Diſtance will enlighten Objects vaſtly diſtant.

2. If C B, the Diſtance of the Radiant C T from the concave *Speculum* B; be leſs than a fourth part of its Diameter, its Image will be E F (by *Prop.* IV and VIII) placed beyond the *Speculum*, and erect as in a plane *Speculum*, and which approaches to the *Speculum* as the Object approaches to it; and ſo on the contrary. After the ſame manner the Image of the Radiant F E, made by a convex *Speculum*, will be C T, whoſe Properties, from what has been ſaid, are eaſily detected: For as the Radiant approaches to the *Speculum*, the Image likewiſe approaches to it; but when that recedes to a Diſtance even infinite, the Image

will ſtop at the middle Point between the *Speculum* and its Center.

3. Suppoſe the Rays of each Point of the Radiant were ſo inflected as to be about forming the Image C T, but are hindered from it by the Intervention of the convex *Speculum* B D. From what has been ſaid above, it is plain that theſe Rays, after Reflexion from the *Speculum*, will make the Image F E; whoſe Properties, Situation, and Figure, with reſpect to the Object C T (if it may be called an Object) are eaſily found by the foregoing Rules.

In the preceding, and all other Caſes, the *Ratio* of the Image to the Object is given, becauſe they appear under the ſame or equal Angles from the Center of the *Speculum*. But this will be more plain by the following *Theorem*.

PROP. IX. THEOR. III.

THE *Radiant and its Image made by the* Speculum, *are ſeen from the* Vertex *of the* Speculum *under equal Angles.* Fig. 14, 15.

From A, the Center of the *Speculum*, upon the Radiant H F, let fall the Perpendicular A E (which, by *Prop.* VIII. will be likewiſe perpendicular to the Image *fh*) meeting the *Speculum* in the *Vertex B*. Join the right Lines *BF*, *BH*, *Bf*, *Bh*. I ſay the Angles *HBF*, *bBf*, are equal. Join *Ff*; this will paſs through the Center *A*, becauſe by Suppoſition the Image of any Point is placed in the *Axis* of Radiation. After the ſame manner the Points *HA* and *b* are placed in the ſame right Line. From the Nature of the Image, *BE* is to *EA*, as *BC* to *CA*: therefore *BE* is to *BC*, as *EA* to *CA*. But becauſe of the equiangular Triangles, *AEF*, *ACf*, *EA* is to *CA*, as *EF* to *Cf*. Wherefore *BE* is to *EF*, as *BC* to *Cf*. And ſince the Angles *BEF*, *BCf* are right, the Angles *EBF*, *CBf* will be equal. After the ſame manner *EBH*, *CBb* are equal: Whence *HBF*, *bBf* are equal. *Q. E. D.*

COROL.

The radiant Line *FEH*, and its Image *fCb*, are to one another as their Diſtances from the *Vertex*, *BE*, *BC*. And if the Radiant be a Surface, it will be to its Image in a duplicate Proportion of thoſe ſame Diſtances. From whence, if the Diſtance of the Radiant and its Image from the *Speculum* be given, the Proportion of the Image to the Radiant is alſo given. In like manner, if the Image of the Radiant *C* made by ſeveral *Specula* be given, we ſhall have the Proportion of the primary Radiant to the laſt Image. It follows farther, that the Line *FH*, and its Image *fh*, will be cut in the ſame Proportion by the right Line *BE*, joining the *Vertex* and Center; or that *FE* is to *EH*, as *fC* to *Ch*.

PROP. X. PROB. VII.

TO find ſuch a Poſition of the radiant Body, that its Image made by the Speculum *may be equal to any given Figure ſimilar to the Radiant: Or, which comes to the ſame thing, that the Radiant and its Image made by the* Speculum *may be in a given* Ratio. Fig. 16.

Let the *Speculum BD* be given, whoſe Semidiameter is *BA*; and let the given *Ratio* be *BA* to *AM*. If the Radiant be a Line, biſect *BM* in *E*, and by ſome of the preceding Propoſitions, find the *Focus C*, correſponding to the *Focus E*. I ſay, *C* is the Place of the radiant Line; becauſe, from the Relation of the Points *C* and *E*, *BC* is to *CA*, as *BE* to *EA*; *BC* will be to *BC* + *CA*, as *BE* to *BE* + *EA*: that is, becauſe *BE* is equal to *EM*, *BC* will be to *BA*, as *BE* to *AM*; and inverting the Proportion, *BC* is to *BE*, as *BA* to *AM*. But (by *Corol. Prop.* IX) *BC* is to *BE*, as the radiant Line placed in *C*, to which *BC* is perpendicular, is to its Image in *E*: Therefore the radiant Line in *C* is to its Image made by the given *Speculum* *BD* at *E*, as *BA* to *AM*; that is, in the given *Ratio*. *Q. E. D.*

But if the Radiant be a Surface, in the room of *AM* in the preceding Conſtruction you muſt take a right Line, to which *BA* is in a *Ratio* ſubduplicate of that which it bears to *AM*.

THE ELEMENTS OF DIOPTRICS.

CAtoptrics being finiſhed, we proceed to Dioptrics; by the Aſſiſtance of which we are ſtill furniſhed with more Inſtruments, and ſuch as are fitter for Uſe, and admitted to a nearer View of the Secrets of Nature. For Glaſs is eaſier brought to a due Figure than Metal, and preſerves it longer: Nor does a *Lens* ſuffer ſo much Loſs in its Poliſh, from any Injuries of the Air, as a *Speculum* does. And it is difficult to make a *Speculum* that ſhall reflect ſuch ſtrong and cloſe Rays as a Glaſs *Lens* tranſmits.

But, beſides theſe phyſical Difficulties, the Cauſes of which are not to be ſought from Dioptrics, a *Lens* is preferable in Practice to a *Speculum*, for the following Reaſon: Becauſe a Fault in a *Speculum* produces an Error in the reflected Ray ſix times greater than an equal Fault in a *Lens*, when the Ray paſſes out of Air into Glaſs; and four times greater, when the Ray paſſes out of Glaſs into Air: For, by *Theor*. I. the Error of Incidence produces an Error twice as great in the reflected Ray, but near thrice leſs in a refracted Ray out of Air into Glaſs; and twice leſs in a refracted Ray out of Glaſs into Air; as will appear at *Schol*. 2. *Prop*. XI.

In Dioptrics, as before in Catoptrics, we ſuppoſe the Eye placed in the *Axis* of Radiation; becauſe this Poſition, where only *Lenſes* are uſed, is eaſieſt obtained: But where *Specula* and *Lenſes* are mixed together, that *Axis* (by *Corol*. 1. *Prob*. 1.) may be inclined in any given Angle.

PROP. XI. THEOR. IV.

A Ray of Light, at a plane Surface of a Medium *of different Denſity, is ſo refracted, that the* Media *remaining the ſame, the right Sine of the Angle of Incidence will, in all Inclinations, bear the ſame Proportion to the right Sine of the refracted Angle.* Fig. 17.

It is ſufficient for a Mathematician to have proved this *Theorem* by Experiment; ſo that from this laid down he may demonſtrate the Properties of a given Figure in refracting, or inveſtigate a Figure whoſe Laws of refracting are given: Yet, for the ſame Reaſons that induced us to demonſtrate a like Catoptrical *Theorem*, we ſhall give the Demonſtration of this.

For this Purpoſe, different Perſons have uſed different, and quite contrary, *Poſtulata*. *Cartes* taking up the ſame *Theorem* (though in other Caſes he affirms the Propagation of Light to be inſtantaneous) would have it in his *Dioptrics*, that a Ray of Light is carried with a greater Celerity through a denſer *Medium*, as Water or Glaſs, than through one leſs denſe, as Air. This appeared too groſs to *Barrow*, *Fermat*, and others, who went into a contrary and more probable Opinion. Moreover, *Barrow* and *Maignanus* believed, that ſome certain Thickneſs of the Ray of Light, however ſmall (which *Cartes* neglected as inconſiderable) was neceſſary to demonſtrate this *Theorem*. We have followed the Method of Geometers, and compoſed a Demonſtration far enough from any Sect, and depending only upon a very ſimple Property of Light, which we aſſumed in the beginning.

Let the Light flowing from the Point A, and comprehended in the Plane of Inflection between A B and A D, fall upon the right Line B D, the common Section of the Plane of Inflection with the plane Surface of the *Medium* B C E D of different Denſity. (We may juſtly conſider this by it ſelf; becauſe the Rays which compoſe it, and their refracted Rays, are, diſtinctly from the other circumfuſed Light, contained within the forementioned Plane by *Ax.* 2.) Let one of the extreme Rays A D be perpendicular to the right Line B D, whoſe refracted Ray D E (whatſoever be the Denſity of the *Medium* B E) will conſequently go directly on in the ſame right Line with A D: But let the other A B, containing a certain Angle with A D, meet the right Line B D, which ſeparates the *Media*, in B; through B draw K B F parallel to A E. Let the aforeſaid Light, contained betwixt the right Lines A B, A D, be ſuppoſed to be propagated in the *Medium* D G, betwixt the right Lines D E and B G continued *in infinitum*: That is, let B G be the refracted Ray of A B, whether the Light paſſes out of a rarer *Medium* into one more denſe, as in *Fig.* 17. or out of a denſer one into a rarer, as in *Fig.* 18. I ſay, the Sine of the Angle A B K will, in any Inclination of the Ray, have the ſame Proportion to the Sine of the Angle G B F.

On the Center B, and with any Diſtance B G, deſcribe an Arch of a Circle meeting the Ray B G in G; from which let fall G N perpendicular to A N, interſecting the right Line K F in F.

The Light which proceeding from A is diffuſed into the right Line B D, will, if B G be put for the Length of the refracted Ray, be diffuſed within the new *Medium* D G into the right Line G N: For the Spaces into which the forementioned Light is diffuſed, muſt be computed by theſe right Lines parallel to the right Line B D, becauſe at B D it loſes its former Degree of Diffuſion, and after that enjoys a new one. The right Line B D which divides the *Media* remaining the ſame, ſuppoſe the *Medium* D G, in which A B is refracted into B G, were taken away, and another *Medium* D C of a different Denſity ſubſtituted in its Place, in which the Ray A B is refracted into B C, *Ex. gr.* meeting the Circle in C, through which draw the right Line C E parallel to B D, meeting A N in E, and K F in M. The right Line C E will be the Space into which the Light is diffuſed within the *Medium* D C, the Length of the refracted Ray B C continuing ſtill the ſame. If from the right Lines G N, C E, into which the Light is diffuſed, you take away the right Lines F N, M E, which depend not at all upon the *Facilities* of the *Media*, that is, their *Readineſs in diffuſing Light* (for the Length and Poſition of the Ray A B remaining the ſame, the forementioned right Lines F N, M E remain the ſame and invariable, howſoever the *Facilities* of the *Media* D G, D C are augmented, diminiſhed, or even annihilated) there will remain G F, C M for the genuine Effects of the *Media* D G, D C reſpectively. Therefore ſince all Things that reſpect the Refraction of the Ray A B from the *Medium* D K, to *Media* of different Denſities D G and D C, excepting only the Denſities of thoſe *Media*, are made the ſame (namely, the *Medium* D K, out of which the Light paſſes the ſame, the Angle of Incidence A B K of the Ray A B the ſame, and the Length of the refracted Ray within the ſecond *Medium* D G, or D C the ſame) the forementioned right Lines G F, C M will have the ſame Proportion with the *Facilities* of the *Media* D G, D C which produced them: For all Effects are proportional to their Cauſes.

Now if the *Medium* D C, in which the Ray A B is refracted into B C, be the ſame with the *Medium* D K, in which Caſe the refracted Ray B C (if it may be called ſo) of the Ray A B will be in a direct Line with the incident one A B; G F will be yet to C M as the Facility of the *Medium*

D G to the Facility of the *Medium* D C, that is, by Conſtruction, to the Facility of the *Medium* D K. Moreover, C M is the Sine of the Angle C B M, or the Angle A B K, which is the Angle of Incidence of the Ray A B; and G F is, to the ſame *Radius*, the Sine of the Angle G B F, that is, of the refracted Angle of the ſame Ray: Wherefore the Sine of the Angle of Incidence is to the Sine of the Angle of Refraction, whatſoever be the Inclination of the Ray A B to the refracting Plane (for the Angle D A B, to which A B K is equal, is taken at Pleaſure) as the Facility of the *Medium* D K to the Facility of the *Medium* D G, inflecting the Ray A B into B G. But the *Media* being ſuppoſed the ſame, the Denſities of the *Media*, and their *Facilities* ariſing from thence, and conſequently the *Ratio* of theſe will remain the ſame: Therefore in every Inclination of the Ray the *Ratio* of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction remains the ſame. *Q. E. D.*

This *Theorem* is thus ſhortly demonſtrated in the Geometric Phraſe. The Poſition of the incident Ray remaining the ſame, the refracted Ray remains the ſame, whatſoever be the Length of the incident Ray A B: Therefore no reſpect is to be had to its Length, or the right Line A B is to be taken for nothing. Therefore the right

Line F N, which bears a certain Proportion to A B, vaniſhes at the ſame time: Whence the right Line G N, into which the Light is diffuſed at a given Diſtance B G within the *Medium* D G, is changed into the only and determinate right Line G F; which is conſequently appointed by Nature for the Meaſure of the Facility of the *Medium* D G, to which it owes its Riſe. *Q. E. D.*

But if in the Paſſage of a Ray from a denſer *Medium* into a rarer (*Fig.* 18.) its Inclination be ſuch, that G F, which is to C M in the Proportion of the *Facilities* of the given *Media*, ſhould exceed the *Semidiameter* B G; then the refracted Ray of A B will be nothing (after the manner of an impoſſible Caſe in a *Geometrical Problem.*) But that which ſhould have been the refracted Ray will not enter the rarer *Medium* D G, but will be reflected from its Surface, according to the Law of *Theor.* I. in which Caſe the Law of this preſent *Theorem* (as being more univerſal) does neverthel eſs take place.

The converſe of this Propoſition is ſufficiently manifeſt; namely, that the refracted Ray of any Incident is truly aſſigned, when the Angle of Incidence is to the Angle comprehended by the aſſigned right Line, and another drawn perpendicular to the refracting Surface, in the Proportion of the

Facilities of the given *Media*, which is the Meaſure of the Refraction between them: For the refracted Ray of the Incident propoſed can be no other than that aſſigned.

The Invention of the preceding *Theorem*, which is the principal one in *Dioptrics*, is commonly attributed to *Carteſius*, tho' it was known to *Willebrord Snellius*, who was dead ten Years before *Cartes*'s *Dioptrics* were publiſhed: For *Voſſius*, in his Treatiſe *De Natura & Proprietate Lucis*, publiſhed at *Amſterdam* in the Year 1662, Page 36, tells us, that it appeared from *Snellius*'s Papers, which himſelf had ſeen, that he had found out, that the Proportion between the Secants of the Angles, which are the Complements of the Angle of Incidence, and the Angle of Refraction to right ones, is conſtantly the ſame: But it is known that the Secants of Angles are reciprocally, as the Sines of their Complements to right ones; becauſe the *Semidiameter* is a mean Geometric Proportional between them.

Since we have happened to ſpeak of Secants, it is worth the taking notice how near the inquiſitive KEPLER was towards finding out this *Theorem*; who, at *Prop.* V. and VI. *Paralipom. in Vitellionem*, lays down theſe Secants for the reſpective Meaſure of Refractions.

Corol. 1.

If B G be the refracted Ray of the Incident A B, every thing elſe remaining the ſame, B A will be the refracted Ray of the Incident G B: For the *Facilities* of the *Media*, by which the Sines of the Angles are governed, remain the ſame.

Corol. 2.

From the Demonſtratien of this *Theorem* it follows, that if the *Ratio* of A to B meaſure the Refraction between the *Media* A and B, and the *Ratio* of A to C the Refraction between the *Media* A and C; the *Ratio* of B to C will be the Meaſure of the Refraction between the *Media* B and C. Which is alſo manifeſt, by ſuppoſing the Thickneſs of the common *Medium* A, between the parallel Planes of the other two, to vaniſh: For by this means, the Angles remaining the ſame, that Part of the Ray, propagated through the three *Media*, which is in the *Medium* A, vaniſhes.

Corol. 3.

Hence it likewiſe follows, that if the Poſition of the Ray A B remaining the ſame, the Poſition of the inflecting right Line B D be changed by an Angle, ſuppoſe E, the Poſition of the refracted Ray B G will alſo be changed by an Angle *e*, which is to E as the Difference of I and R to I; (the *Ratio* of the Quantities I and R being ſuppoſed the ſame with the *Ratio* of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction; and the Angles ſo ſmall, that they may be looked upon to have the ſame Proportion with their Sines) and the Angles *e* and E are ſituated on the ſame Sides of the right Lines B G and B D reſpectively, when I exceeds R; but on contrary Sides when R exceeds I. After the ſame manner, from this Propoſition, may we judge of the Change the Angle G B F will undergo, when the Angle A B K is changed by any other Cauſe whatever.

SCHOLIUM I.

The aforeſaid Law of Refraction is confirmed from the Wiſdom of Nature, always acting by the moſt eaſy and expeditious Methods; which we before found in *Catoptrics* to be a legitimate *Axiom*; namely, to the Light proceeding from A to G, ſuch a Point of Incidence B is aſſigned, that it may perform its Journey A B G in the *leaſt Time* poſſible. Which

Point of Incidence may be found by the Help of any Method (*Fermat*'s for Inſtance) which determines a *Maximum & Minimum*, but moſt eaſily after the Manner following.

PROB.

TWO Points being given in Media *of different Denſities, and the plane Surface dividing the* Media, *whoſe Denſities are likewiſe given, being given in Poſition: To find ſuch a Point in the foreſaid Surface, that a moveable Body proceeding from one of the given Points, through the Point ſought to the other of them, may take up the* leaſt Time *in its Journey.* Plate II. Fig. 1.

Let the given Points be A and G, and let M N be the Surface dividing the *Media* of different Denſities, or rather the common Section of the Plane of Inflexion continued through A and G, with the foreſaid Surface dividing the *Media*: For (by *Ax.* 2.) the Point ſought will be found in that. Let the right Lines I and R expreſs the Velocities of Light in the *Media* DK, CF reſpectively.

Becauſe the Velocity of a moving Body being given, the Time is as the Space run; and the Space being given, the Time is as the Velocity reciprocally; if neither being given, the Time will be in a compound Proportion of the Velocity reciprocally, and the Space directly. Therefore the Time in which the right Line A B is run will be expreſſed by R × A B, and the Time in which B G is run, by I × B G; and conſequently the Time in which the whole Journey A B G is performed, which is made up of both of them, will be expreſſed by the Quantity R × A B + I × B G; which muſt be the *Minimum.*

From the Poſition of the Points A and G, and of the right Line M N being given, the right Lines A D, G C, and D C, which we ſhall call *a*, *c*, and *d* reſpectively, are given in Magnitude. Call C B *x*. Whence B D = $d - x$, A B = $= \sqrt{d^2 - 2dx + x^2 + a^2}$, and B G = $\sqrt{c^2 + x^2}$. Therefore $R \times \sqrt{d^2 - 2dx + x^2 + a^2} + I \times \sqrt{c^2 + x^2}$ muſt be a *Minimum.*

Now this is (as Mathematicians know very well) when the *Fluxion* of $R \times \sqrt{d^2 - 2dx + x^2 + a^2} + I \times \sqrt{c^2 - x^2} = 0$. But by the *Method* of that moſt celebrated Geometer Sir *Iſaac Newtou, for finding the Fluxions of as many Fluents as are involved in any given Equation* (to which alſo the famous *Leibnit's Differential Calculus* relates) which you may ſee at *Cap.* XCV. *Vol.* II. *Operum Mathemat. Walliſii*, The

Fluxion of $R\times\sqrt{d^2-2dx+x^2+a^2}+I\times\sqrt{c^2+x^2}$ is $R\times\frac{\overline{-2dx+2xx}}{2\sqrt{d^2-2dx+x^2+a^2}}+\frac{I\times 2xx.}{2\sqrt{c^2+x^2}}$ Therefore making this equal to nothing, we have

$$\frac{I\times x}{\sqrt{c^2+x^2}}=\frac{R\times\overline{d-x}}{\sqrt{d^2-2dx+x^2+a^2}}.$$

And ſubſtituting, in the room of their Valours, the right Lines themſelves expreſſed in the Scheme, it becomes $\frac{I\times CB}{BG}=\frac{R\times BD.}{AB}$ And if BG=AB, then I×CB=R×BD, or BD. BC: : I. R.

Therefore if the right Line DC be divided in B, ſo that DB may be to BC in the given *Ratio* of I to R, the Light will perform its Journey from A to G (or backward from G to A) in the ſhorteſt or leaſt Time poſſible, by going along ABG. But DB and BC are the Sines of the Angles BAD and BGC; that is, of the Angles ABK, GBF; namely, of the Angle of Incidence, and the Angle of Refraction. Wherefore, that the Time of the Paſſage from A to G (or from G to A) may be the *leaſt*, the Ray muſt ſo fall, that the Sine of the Angle of Incidence may be to the Sine of the Angle of Refraction in the *Ratio* of I to R; that is, in the *Ratio* of the Velocities of Light in the foreſaid

ſaid *Media*, or (as is ſhewn before in the Demonſtration of *Prop.* XI.) in the *Ratio* of the *Facilities* of the ſame *Media*, upon which the *Velocities* of the Body moving through them depend.

Since there are in the Plane dividing the *Media* an infinite Number of ſuch Points (two of which are in the right Line M N) that the Light, in paſſing from A to G (or from G to A) through any of them, ſhall take up any given Time exceeding the *leaſt*; there can be no Reaſon aſſigned, why it ſhould paſs through one of thoſe Points rather than another: Therefore it will paſs through none but the only and determinate one of its kind, B.

SCHOLIUM 2.

When a Ray of Light paſſes out of Air into Glaſs, we obſerve the Sine of the Angle of Incidence to be to the Sine of the Angle of Refraction, as 3 to 2 in round Numbers: And in paſſing out of Air into Water, I is to R (which Symbols we ſhall uſe for the future, to expreſs the *Ratio* of the Sine of the Angle of Incidence to the Sine of the Angle of Refraction, which is the Meaſure of Refraction) as 4 to 3. And on the contrary, in the Paſſage of a Ray out of Glaſs into Air, I will be to R as 2 to 3, by *Corol.* 1. From whence the Reaſon is plain, why a Ray in Glaſs, ſtriking upon a Surface of Air, more obliquely than in an Angle of Incidence of about 42 Degrees, does not enter the Air, but is reflected from its Surface: For the right Line G F, which is ſeſquialteral of the Sine of 42°, exceeds the *Radius*; in which Caſe the Light ſhall be reflected, as has been obſerved above.

Moreover, becauſe in the Paſſage of a Ray out of Air into Glaſs, I is ſeſquialteral or $1\frac{1}{2}$ of R, and in its Paſſage out of Air into Water, I is $1\frac{1}{3}$ of R; in its Paſſage out of Water into Glaſs (by *Corol.* 2.) I will be $1\frac{1}{8}$ of R.

And from *Corol.* 3 it follows, that the Error of the refracted Ray is but ſubtriple of the Error of the Surface of Glaſs by which it was occaſioned, and on the ſame Side with it, when the Ray paſſes out of Air into Glaſs; but ſubduple of it, and on the contrary Side, when the Ray paſſes out of Glaſs into Air.

PROP. XII. THEOR. V.

LET BD *be a Surface dividing* Media *of different Denſities; upon which let a Ray fall proceeding from* E, *and be refracted into* D N, *the Surface dividing the* Media *remaining the ſame, but the*

Media *being tranſpoſed ; I ſay the refracted Ray* DC *of the Ray* MD *being placed in the ſame right Line with* ED, *will be in the ſame right Line with the former refracted Ray* ND. Fig. 2.

Draw the right Line ADO perpendicular to the Surface BD in D.

Since DN is the refracted Ray of ED, (by the foregoing) the Sine of the Angle EDA will be to the Sine of the Angle NDO in the *Ratio* of I to R; and if the *Media* be tranſpoſed, the *Ratio* of I to R will be the Meaſure of the Refraction out of the *Medium* BDM into the *Medium* BDE: That is, ſuppoſing MD to be the incident Ray, its refracted Ray will be ſuch, that the Sine of the Angle MDO ſhall be to the Sine of the Angle ADC, as I to R: That is, by what has been juſt now ſhewn, as the Sine of the Angle ADE to the Sine of the Angle NDO. Therefore ſince the Angles EDA, MDO are equal, and conſequently their Sines, the Sines of the Angles ADC, NDO will be likewiſe equal, and conſequently the Angles themſelves. Wherefore ND and DC lie in the ſame right Line. *Q. E. D.*

COROL.

If Rays diverging from E, after Refraction at the Surface BD, diverge from the Point C, the *Media* being tranſpoſed, and the Surface dividing the *Media* remaining the ſame, Rays converging towards E will, after their Refraction at B D, converge towards the Point C.

Therefore all that can be demonſtrated of Rays diverging, may equally be applied to converging ones. Wherefore for the future we ſhall only ſpeak of Rays diverging, leaving thoſe converging to be determined by this *Theorem.*

PROP. XIII. PROB. VIII.

F*Rom the* Focus *of diverging Rays being given, to find the* Focus *of the ſame Rays after their Refraction, at a plane Surface of a* Medium *of different Denſity, with reſpect to an Eye placed in the* Axis. Fig. 3, 4.

Through E, the given *Focus* of diverging Rays, draw the right Line E B perpendicular to the plane Surface B D of a *Medium*, either denſer or rarer; in this take the Point C ſuch, that C B may be to E B, as I to R. I ſay C is the *Focus* ſought.

Let fall any Ray E D. Through D draw A O parallel to E B. Join C D, and produce it to F.

If the Ray E D be near enough to the right Line E B (for we ſpeak here only of thoſe Rays that fall near the *Axis*, ſince theſe only do, after Refraction, affect the Eye placed, by Suppoſition, in the *Axis* of Radiation produced; thoſe that fall more obliquely paſſing by the Pupil of the Eye, and conducing nothing at all towards diſcerning the Point E, as we have before obſerved in *Catoptrics*) E D will be nearly equal to E B, and C D to C B. Wherefore C D is to E D, as I to R: But C D is to E D, as the Sine of the Angle B E D, or its equal E D A, to the Sine of the Angle B C D, or its equal O D F: Therefore the Sine of the Angle E D A is to the Sine of the Angle O D F, as I to R. But the Angle E D A is the Angle of Incidence of the Ray E D: Therefore O D F is the reſpective Angle of Refraction; that is, D F is the refracted Ray of the Incident E D. And the ſame may be ſhewn of any other Ray diverging from E; whence C is the *Focus* required. *Q. E. I.*

COROL. I.

From hence, and from *Corol.* 1. *Prop.* XI. it follows, that the ſame Things being ſuppoſed as before, Rays in the *Medium* B D O converging towards C, will, after Refraction, converge towards E.

And if the *Media* were tranſpoſed, it follows from hence, and from *Prop.* XII. that Rays in the *Medium* B D O converging towards E, will, after Refraction, converge towards the Point C; and parallel Rays will, after Refraction, at a plane Surface of any *Medium*, ſtill remain parallel.

Corol. 2.

If BDA be Water, and BDO Air, CB will be to E B, as 3 to 4: From whence it happens, that Water appears a fourth part leſs deep than it really is. But if B D A be Glaſs, E B will be 1½ of the right Line C B.

Scholium.

What is demonſtrated of any incident Ray, namely, that its refracted Ray D C produced backwards, till it meets with EB, is to the Incident E D as I to R, is alſo true of the perpendicular Incident E B: For its refracted Ray B C produced backwards, though as to its Poſition it be the ſame with the Incident, yet as to its Length it is to it in the very ſame *Ratio* of I to R. And becauſe this Ray paſſing through the Center of the Eye, and the others that are neareſt it are the only ones that affect the Senſe; from thence it is that, with reſpect to the Eye, C is the *Focus* of the Rays diverging from E, and refracted at B D. This Conſideration extends likewiſe to what follows.

Prop. XIV. Prob. IX.

To find the Focus *of parallel Rays falling upon a ſpherical Surface of a* Medium *of different Denſity, after Refraction.* Plate II. Fig. 5, 6.

Let the ſpherical Surface, whoſe Center is A, be expreſſed by the Circumference of the Circle B D; through A draw the *Axis* A B parallel to the incident Rays, meeting the Circle in the *Vertex* B; in which take the Point C ſuch, that B C may be to A C, as I to R. I ſay C is the *Focus* required.

Let any Ray fall at D; draw the right Lines A D, C D, and produce them. The Sine of the Angle B A D, or its equal E D O, is to the Sine of the Angle C D A (or its Complement to two right ones) as C D to A C, or (the Arch D B vaniſhing, for Reaſons before often mentioned) as C B to A C; that is, by Conſtruction, as I to

R: But E D O is the Angle of Incidence of the Ray E D; therefore D C is its refracted Ray. After the ſame manner it may be demonſtrated, that any other of the parallel Rays will, after Refraction, paſs through C: Therefore C is their *Focus*. *Q. E. D.*

COROLLARIES.

Hence, and from *Corol.* 1. *Prop.* XI. and *Corol. Prop.* XII. it follows;

1. That Rays in Air parallel to the *Axis*, after Refraction at a *Spherico-convex* Surface of Glaſs, converge to a Point whoſe Diſtance from the *Vertex* is equal to three Semidiameters of the Sphere.

2. That Rays in Glaſs, diverging from a Point diſtant from a *Spherico-concave* Surface of Air by three Semidiameters, do, after Refraction, become parallel to a right Line drawn through the radiant Point and the Center of the Sphere.

3. That Rays in Glaſs parallel to the *Axis*, after Refraction at a convex ſpherical Surface of Air, diverge from a Point whoſe Diſtance from the *Vertex* is equal to the Diameter of the Sphere.

4. That Rays in Air, converging towards a Point diſtant beyond a concave Glaſs by the Diameter of the Sphere, after Refraction at the concave Surface of Glaſs,

become parallel to a right Line drawn through the Center of the Sphere and the forementioned Point.

5. That Rays in Air parallel to the *Axis*, after Refraction at a concave Surface of Glaſs, diverge from a Point at three Semidiameters Diſtance from the *Vertex*.

6. That Rays in Glaſs, converging to a Point three Semidiameters diſtant beyond a *Spherico-convex* Surface of Air, after Refraction, become parallel to a right Line drawn through the Center of the Sphere and the forementioned Point.

7. That Rays in Glaſs parallel to the *Axis*, after Refraction at a *Spherico-concave* Surface of Air, converge to a Point diſtant from the *Vertex* by a Diameter of the Sphere.

8. That Rays in Air, diverging from a Point at a Diameter's Diſtance from a Sphere of Glaſs, after Refraction at a convex Surface of Glaſs, become parallel to a right Line drawn through the forementioned Point and the Center of the Sphere.

SCHOLIUM. *Plate* II. *Fig.* 7.

The generating Circle of the Sphere B D has the ſame Degree of Curvity with an *Ellipſis* paſſing through B; by whoſe Rotation a *Spheroid* being made of a denſer *Medium*, collects Rays in the ambient *Medium*

Medium parallel to the right Line A B, exactly at C: For we all know, that B G, the greater *Axis* of that *Ellipſis*, ought to be to E C, the Diſtance of the *Foci*, as I to R. Suppoſing then the ſame Conſtruction with that at *Schol. Prop.* IV. B G is to E C as K C to H C; that is, as H C to C L; that is, as K C + H C to H C + C L, or to H C + C K — L K; that is, as B C to B C — half the *Latus Rectum*, belonging to the *Axis* B G (for L K has, at *Schol. Prop.* IV. been ſhewn equal to half the forementioned *Latus Rectum*) But by the Conſtruction of this *Prop.* the Situation of the Point C is ſuch, that B C is to A C as I to R; that is, as B G to E C: Therefore B C is to A C, as B C to B C — half the *Latus Rectum*, belonging to the greater *Axis*: Wherefore A C is equal to B C — half the forementioned *Latus Rectum*; and conſequently A B is equal to half the *Latus Rectum*. Wherefore the Circle B D is equally curve in B with the *Ellipſis* B K G (by *Corol.* 1. *Prop.* XX. *Lib.* 1. *Vinc. Viviani de Maximis & Minimis.*) Whence it is, that a Sphere generated by the forementioned Circle performs in refracting very nearly the ſame thing which an oblong *Spheroid*, generated by the Rotation of an *Ellipſis*, does exactly; namely, that Rays in a rarer ambient *Medium*, parallel to the right Line B G, after Refraction at its Surface, may converge to the Point C.

After the ſame manner, by the Help of *Schol. Prop.* VI. it may be ſhewn, that the Circle B D is at the *Vertex* equally curve with an *Hyperbola* paſſing through B, and generating a Conoid; which being of a rarer *Medium*, does ſo refract Rays in a denſer ambient *Medium* parallel to the right Line A B, that they ſhall afterwards diverge from the Point C: Or, which returns to the ſame thing (by *Prop.* XII.) being of a denſer *Medium*, does ſo refract Rays in that ſame denſer *Medium*, parallel to the right Line A B, at their Entrance into the rarer ambient *Medium*, that they ſhall afterwards converge to the Point C.

PROP. XV. PROB. X.

THE Focus *of Rays diverging and falling upon a given ſpherical Surface of different Denſity, to find the* Focus *of the ſame Rays after Refraction.* Plate II. Fig. 8, 9, 10, 11, 12, 13, 14, 15.

Let the ſpherical Surface be expreſſed by the Circumference of its greateſt Circle B D. Through the given *Focus* and the

Center of the Sphere draw the right Line *EA* meeting the Circumference in *B*; in which take the Point *C* ſuch, that the *Ratio* compounded of the *Ratio* of *EA* to *AC*, and of *CB* to *BE*, may be equal to the *Ratio* of *I* to *R*. I ſay *C* is the *Focus* required.

We ſhall make one Demonſtration ſerve for all the eight principal Caſes of this *Prop.* expreſſed by ſo many ſeveral Figures; the four firſt of which, ſuppoſe *I* greater than *R*, the others leſs, and which differ from one another, according as the given *Focus* is ſituated on this or that Side of the *Focus* of parallel Rays, or according as the refracting Surface is convex or concave.

Let fall any Ray at pleaſure, *ED* proceeding from *E*. Join *AD*, *DC*; and through *C* draw a right Line parallel to *AD*, meeting the right Line *ED* in *H*.

The Point *D* almoſt coinciding with *B*, *CD* and *CB*, as likewiſe *ED* and *EB*, are almoſt equal. Wherefore the *Ratio* of *I* to *R* is equal to the *Ratio* of *EA* to *AC*, and the *Ratio* of *CD* to *ED* together; that is (becauſe of *AD* and *CH* parallel) to the *Ratio* of *CD* to *ED*, and the *Ratio* of *ED* to *DH* together, or to the *Ratio* of *CD* to *DH*: But *CD* is to *DH* as the Sine of the Angle *DHC*, or its Complement to two right ones; *ADH*, or *EDO*, is to the Sine of the Angle *DCH*, or of the Angle *ADC*, or in ſome Caſes its Complement to two right ones. Moreover, the Angle *EDO* is the Angle of Incidence of the Ray *ED*: Therefore as *I* to *R*, ſo the Sine of the Angle of Incidence of the Ray *ED* to the Sine of the Angle *ADC*. Wherefore (by *Prop.* XI.) *DC* is the refracted Ray belonging to the incident one *ED*; and ſince *ED* is taken at Pleaſure, 'tis plain the *Focus* of all the Rays proceeding from *E* will be *C*. *Q. E. I.*

If the *Media* propoſed were Air and Glaſs, and the Ray paſſed out of Air into Glaſs (as in the four firſt Figures) the *Focus C* will be very eaſily found; namely, if the third part of the right Line *EA* be to *AB*, as *EC* to *BC*: For trebling the Antecedents, *EA* is to *AB* (or *AD*) as 3 *EC* to *BC* (or *DC*): But *EA* is to *AD*, as the Sine of the Angle *EDO* to the Sine of the Angle *DEA*; and 3 *EC* is to *DC* as thrice the Sine of the Angle *EDC* (or *HDC*) to the Sine of the Angle *DEA*: Wherefore the Sine of the Angle *EDO* is triple of the Sine of the Angle *HDC*, and conſequently (in theſe very ſmall Angles, which have the ſame *Ratio* with their Sines) $1\frac{1}{2}$ of the Sine of the Angle *ADC*. Therefore *DC* (or *De* lying in the ſame right Line with it) is the refracted Ray of *E D* paſſing out of Air into Glaſs.

When the Ray paſſes out of Glaſs into Air (as in the four laſt Figures) the *Focus* will be found from the ſame Principles, by taking the Point *C* ſuch, that half the right Line *E A* may be to *A B*, as *E C* to *B C*.

Corol. 1.

From this *Prop.* and XII. and *Corol.* 1. *Prop.* XI. it will be eaſy, from the given *Focus* of Rays converging, and falling upon a ſpherical Surface of a *Medium* of different Denſity, to find their *Focus* after Refraction.

Corol. 2.

Hence, from the *Foci E* and *C*, and the *Vertex B* being given, we may find *A* the Center of the Sphere, and thence the refracting Sphere it ſelf; by taking the Point *A* ſuch, that *E A* may be to *A C* in a *Ratio* compounded of the *Ratio*'s of *B E* to *B C*, and *I* to *R*: Or, from the *Foci* and Center being given may be found the *Vertex*. The like Problem concerning ſpherical *Specula*, may (by the *Correſpondent Prop. in Catoptrics*) be ſolved with equal Eaſe, and more elegant

Geometrical Conſtructions from thence deduced.

COROL. 3.

The Geometrical Conſtruction of this *Prob.* X. is eaſily deduced from what has been premiſed, *Plate* II. *Fig.* 16. Every thing elſe remaining as before, through *B* and *A* draw the right Lines *BQ*, *AN* parallel, meeting the right Line *EN* drawn from *E* as you pleaſe in *F* and *N*: Make *AN* to *AM*, as *I* to *R*. Join the right Line *FM*, and produce it till it meets the *Axis EA* in *C*. This will be the *Focus* required.

Join *CN*, meeting the right Line *BF* in *Q*; to this, through *F*, draw parallel the right Line *FH*, interſecting the right Line *EA* in *H*.

For *EA. EB : : (EN. EF : : EC : EH : : EC—EA. EH—EB : :) AC. BH. & permutando EA. AC : : EB. BH.* Therefore *CB. BE, + EA. AC : : (CB. EB, + EB. BH : : BC : BH : : BQ. BF : : AN. AM : :) I. R.* Whence *C* is the *Focus* required.

This ſame Conſtruction will ſerve (only changing the Order of the Points *E*, *B*, *A*, *C*, *M* and *N*) if *R* exceed *I*, or if the Rays converge, or are parallel; in which Caſe it will be changed into that of *Prob.* IX.

Or if the Concavity of the refracting Surface look towards the *Focus* E, or if B D be plane, in which Caſe it changes into that of *Prob.* VIII. For when the right Line A M N (divided as above) is infinitely diſtant from B Q, the right Line F M will be the ſame in Poſition with a right Line joining the Point F with a Point dividing B E, after the ſame manner as A M is divided in N, ſuppoſing B E to be homologous to A M; becauſe the Tangents of Angles are reciprocally as the Tangents of their Complements to a right Angle.

After the ſame manner, if of the four Points A, B, C, and E, any other three be given, *Ex. gr.* E, B, and C, the fourth A will be determined; that is, the ſpherical Surface paſſing through the given Point, and changing the *Focus* E into C, will be determined; *& vice verſa.*

SCHOLIUM.

Since there is no Spheroid or Conoid generated by the Rotation of a conic Section round its *Axis*, and conſiſting of a *Medium* of different Denſity from the ambient *Medium*, which can exactly change a given *Focus* of Rays into another given one by Refraction at a ſingle Surface; it follows, that this Property of performing the Thing propoſed pretty nearly, does therefore belong to a ſpherical Surface; becauſe the greateſt Circle of that Sphere has the ſame Degree of Curvity with *Cartes*'s *Curve*, (*Lib.* II. *Geometriæ*) by whoſe Rotation are made Surfaces, which, ſeparating *Media* of different Denſity, anſwer the Problem exactly. But if the Condition of that equicurve Circle, and of the given *Foci* lying on the ſame Side of the Center, be ſuch, that the *Semidiameter* of the Circle be a Middle proportional between the Diſtances of the *Foci* from the Center of the Circle; and one of theſe Diſtances be to the *Semidiameter* in a *Ratio*, meaſuring the Refraction between the given *Media*, that Curve of *Cartes* is changed into the Circumference of a Circle. In this Situation of the *Focus* a Sphere of given Denſity, generated by the Rotation of a Circle, will ſo refract Rays proceeding from one of the *Foci*, that they ſhall all afterwards diverge preciſely from the other of them.

PROP. XVI. PROB. XI.

ONE Focus *of a given* Lens *being given, to find the other.*

A *Lens* is a tranſparent Body, of a different Denſity from the ambient *Medium*, and terminated by two Surfaces, either ſpherical, or plane and ſpherical. A right Line perpendicular to both its Surfaces is called the *Axis* of the *Lens*. The Points where the *Axis* interſects each Surface are called the *Vertices*, either the *Vertex* of *Incidence*, or the *Vertex* of *Emerſion*, according as it lies in that Surface upon which the Rays firſt fall, or out of which they again emerge. The *Thickneſs* is the Diſtance between the *Vertices*.

The Terms being thus explained, the *Focus* required may, by means of a Calculation (which in Practice is to be preferred to the niceſt Conſtructions) grounded upon the foregoing Propoſitions, be eaſily determined, and Canons (which, becauſe of their Difficulty to be remembered, are here omitted) eſtabliſhed, by making the Calculation general. As if from the *Focus* before Incidence being given, it were required to find the *Focus* after Emerſion: Firſt, the *Focus* of the Rays, after their Refraction at that Surface of the *Lens* upon which they firſt fall, muſt be found; and this is done by *Prop.* XIII. if the Surface of the *Lens* be plane; but if it be ſpherical, and the Rays parallel, by *Prop.* XIV. and by *Prop.* XV. if they be diverging or converging. And having thus got the *Focus* of the Rays, after Refraction at this firſt Surface, that is, while they are paſſing through the *Lens*, which is likewiſe called the *Focus of Tranſition*, after the ſame manner their *Focus*, after Refraction at the ſecond Surface of the *Lens*, or rather at the Surface of the ambient *Medium* contiguous to this ſecond Surface, will be found; that is, their *Focus* after Emerſion from the *Lens*. *Q. E. I.*

If there were more than one *Lens*, we muſt proceed after the ſame manner with every one of them.

By the like Method, from the *Focus* made by the Help of one or more given *Lenſes* being given, the *Focus*, before Incidence is found, or from the optical Machine, being given, the Diſtance of the Object is determined.

COROL.

If the Geometrical Conſtruction of this Problem be deſired, it is eaſily deduced from *Corol.* 3. of the foregoing *Prop.* by aſſuming the Conſtruction there given for one Surface of a *Lens*, and repeating it for the other, *Fig.* 17. For the right Lines E N, B F, A N, and F M, being drawn according to the Directions of that *Corol.* 'tis plain the right Line F M will tend towards the *Focus* of Tranſition of thoſe Rays, whoſe *Focus* before Incidence was E. If therefore this meets with the parallel right Lines, drawn through *a*, the Center of the hindmoſt Surface, and *b*, the *Vertex* of Emerſion, in *n* and *f*, and *a n* be taken to *a m*, as I to R at the Egreſs out of the *Lens*; (that is, as R to I at the Ingreſs into the *Lens*) the joined right Line *m f* will meet the *Axis* B *b* in *e*, the *Focus* required, after Refraction at both the Surfaces of the *Lens*: For the right Line *m f* is in the ſame Condition, with reſpect to the Emerſion of the Rays from the *Lens*, in which M F was with reſpect to their Immerſion.

In like manner, if of the ſix Points, A, B, E, *a*, *b*, *e*, any other five be given, the ſixth may be determined: For Example; From the two *Foci*, the Thickneſs of the *Lens*, and one Surface being given, the other Surface may be diſcovered.

This is the Conſtruction which *Barrow* received from a Friend, and placed, without any Demonſtration, at the End of *Lect.* XIV.

PROP. XVII. THEOR. VI.

IF a plane radiant Surface ſends out Rays upon any plane or ſpherical refracting Surface, the Rays proceeding from every

every Point of the radiant Surface will, after Refraction, have their reſpective Foci *very nearly in one and the ſame Plane, parallel to the radiant Plane.* Plate II. Fig. 18.

Let any refracting Surface be expreſſed by B D, whoſe Center is A, and a plane radiant Surface by the right Line E F. From A upon E F let fall the Perpendicular A E meeting B D in B. Find C, the *Focus* of thoſe Rays after Refraction, whoſe *Focus* before Incidence was E; through which draw the Plane C T parallel to the Plane F E. I ſay, the *Foci* of Rays proceeding from every Point of the Plane E F (or the principal Image of the Plane E F, which is made up of the Images of every one of thoſe Points, with reſpect to an Eye placed in the *Axis* of Radiation: For we take no Notice here of the ſecondary Image ſeen by an Eye in any given Situation; which *Barrow* has conſidered in his three laſt *Lect.*) will all be poſited very nearly in the Plane C T. From the Point F, taken at Pleaſure in the radiant Plane, to the Center A, draw the right Line F A, meeting the refracting Surface at D, and the Plane C T in T. D C will be the refracted Ray of the Incident E D; and ſuppoſe the refracted Ray of the Incident F B to be B H, meeting the right

Line D T in H. Becauſe the Angle E A F is ſuppoſed but ſmall, the Arch E D is to be looked upon as a right Line; and a Circle deſcribed on the Diameter B F will paſs through the Points D and E, becauſe of the Angles B E F, B D F being right. Whence the Angles E B F, D B F (namely the Angles of Incidence of the Rays F B and E D) contained in the ſame Segment, are equal; and conſequently the Angles of Refraction, A B H, A D C, will alſo be equal. Therefore, by reaſon of the Angles at A equal, the Triangles B A H, D A C are equiangular; and B A is to A H, as D A to A C; and ſince B A is equal to D A, A H will be likewiſe equal to A C: But becauſe the Angle E A F, or T A C, is very ſmall, A T is very nearly equal to A C; and therefore A H, A T may be looked upon as equal; that is, the *Focus* of the radiant Point F is ſituated very nearly in the Plane C T. And ſince F is taken any how, the ſame will hold true of all the Points of the Plane E F; namely, that their *Foci* will be in the Plane C T. Which is demonſtrated after the ſame manner in any other Caſe whatever. *Q. E. D.*

COROL.

Hence it follows, that the Image of the radiant Plane E F, to which the *Axis* of the *Lens* is perpendicular, is alſo a Plane parallel to the former Plane: For the Image conſtituted in the Plane C T may be conſidered as a plane Surface ſending out Rays upon the ſecond Surface of the *Lens*. But if the Angle E A F be too great, ſo as that A T ſhall much exceed AC, and can by no means be looked upon as equal to it; in that Caſe 'tis eaſy, after the manner of *Corol. Prop.* VIII. to determine whether the Image of the Plane E F will be convex or concave: For Example; It will be concave towards A, if B D, the refracting Surface, be of a denſer *Medium*, and convex, or of a rarer *Medium*, and concave; *& vice verſa*.

PROP. XVIII. THEOR. VII.

THE Image appears from the Vertex *of Emerſion under an Angle equal to that under which the Object appears from the* Vertex *of Incidence.* Plate II. Fig. 19.

Let G B, the *Axis* of the *Lens*, be ſuppoſed produced, and ſtanding perpendicularly on the radiant Plane F E H at E, which (by *Corol.* of the foregoing *Prop.*) is alſo perpendicular to its Image *f* C *h*. Join the right Lines B F, B H, G *f*, G *h*. I ſay, the Angles F B H, *f* G *h* are equal. Out of the innumerable Rays proceeding from the Point F, and after Refraction at the *Lens*, again united at *f*, the Image of this Point, chooſe two, one of which, F B, meets the *Lens* at the *Vertex* of Incidence B, and being there refracted, tends to φ, the *Focus* of Tranſition of that Point; and being again refracted at L, is directed towards *f*: The other, F D, being firſt refracted in D, tends ſtraight on to φ, till it emerges out of the *Lens* at the *Vertex* of Emerſion; where being again refracted, it proceeds towards *f*.

D G B is the Angle of Incidence of the Ray D G, and C G *f* its Angle of Refraction; and L B G is the Angle of Incidence of the Ray L B, which (by *Corol.* 1. *Theor.* IV.) would be refracted into B F, and E B F is its Angle of Refraction. Becauſe the right Lines φ B, φ G (B G, the Thickneſs of the *Lens*, being neglected) become equal, the Sines of the Angles D G B, L B G, which are proportional to theſe, and conſequently the Angles themſelves, D G B, L B G, and therefore their Angles of Refraction, likewiſe C G *f*, E B F, will be equal. After the ſame manner C G *h*, E B H are found equal; therefore the Angles F B H, *f* G *h* are equal: Which is ſhewn after the ſame manner in any other Caſe whatever. *Q. E. D.*

COROL.

Hence it follows, that a radiant Line is to its Image made by a *Lens*, as the Diſtance of that from the *Vertex* of Incidence, to the Diſtance of this from the *Vertex* of Emerſion, or (the Thickneſs of the *Lens* being neglected) as their Diſtances from the *Lens*. But if the Radiant be a Surface, their homologous Lines will ſtill remain in the ſame Proportion; but the Radiant will be to its Image in a duplicate Proportion of thoſe Diſtances. Whence it will be eaſy to determine the Proportion which the *laſt Image* (which is immediately ſeen by the Eye) of an Object, made by the Mediation of one or more *Lenſes*, bears to the Object it ſelf.

From this *Prop.* it follows likewiſe, that a radiant Line and its Image are cut in the ſame Proportion by the *Axis* of the *Lens* produced.

PROP. XIX. PROB. XII.

THE *Poſition of a Radiant in the* Axis *of the* Lens *being given, to determine its Image made by a given* Lens, *with reſpect to an Eye placed in the* Axis *of the* Lens. Plate II. Fig. 20.

Let P C Q D be the *Lens* propoſed; in whoſe *Axis* C D, produced at Pleaſure, ſuppoſe the Radiant to be placed, to the extreme Points of which the right Lines C E, C F do tend. Let *b* be the *Focus* of Rays proceeding from that Point of the Radiant which lies in the *Axis*, after Refraction at both the Surfaces of the *Lens*, found by *Prop.* XVI. Through which draw the Plane *e b f*, to which C D is perpendicular. Then through D the right Lines D *f*, D *e* being drawn parallel to C F and C E, 'tis plain (from the two foregoing *Prop.*) that *f* is the Image of that Point to which the right Line C F is directed, and *e* is the Image of that to which C E is directed. From whence it is manifeſt, that the Image *f b e* will be ſeen by an Eye placed any where in C D produced beyond *b*, and receiving the *Rays* from the reſpective *Points* of the Image diverging.

But it is to be obſerved, that every Point of the Image f b e *does not, like the primary Radiant, ſend forth Rays every way, and into all Parts; but the Rays of each Point conſtitute a Cone oppoſed at the* Vertex *to that Cone, which has the forementioned Point for its* Vertex, *and the* Lens *that refracts the Rays for its Baſe*: Whence, from the Situation of the Eye, and the Diameter of its Pupil being given, it will preſently be known whether the Eye will receive the Rays of any given Point; that is, whether it will ſee that Point. *Which Conſideration muſt always take place likewiſe in Viſion of Images made by a* Speculum.

For Example; If the Radiant be vaſtly diſtant, and the Surface P C Q plane, P D Q convex, and the *Lens* made of Glaſs with Air all around it, D *b* will (by *Corol.* 3. *Prop.* XIV.) be equal to the Diameter of the Sphere P D Q: For the Rays being parallel to B C, paſs unrefracted through the plane Surface P C Q, upon which they fall perpendicularly; and the Image in reſpect to the Radiant is inverted.

But if P C Q be convex, and P D Q plane, C *b* will (by *Corol.* 1. *Prop.* XIII. and *Corol.* 1. *Prop.* XIV.) be equal to the Diameter of the Sphere P C Q, together with a third part of C D, the Thickneſs of the *Lens.* And neglecting the Thickneſs of the *Lens*, as is uſually done in the object *Lens* of a Teleſcope, the Diſtance of the Image of a vaſtly diſtant Radiant, from a Plano-convex *Lens*, is equal to the Diameter of the Sphere.

If the Plano-convex *Lens* P Q were of Water, D *b* would, by *Prop.* XIV. be ſeſquialteral of the Diameter.

If the *Lens* P Q be of Glaſs, and both its Surfaces equally convex, the Thickneſs being neglected, D *b* will (by *Corol.* 1. *Prop.* XIV. and *Prop.* XV.) be equal to the *Semidiameter* of either Sphere. In an entire Sphere of Glaſs, the Image of a very diſtant Radiant will be at the Diſtance of a Quarter of the Diameter behind the Sphere; for in this Caſe the Thickneſs of the *Lens* cannot be neglected: In one of Water this Diſtance equals the *Semidiameter*.

In all theſe and the like Caſes, if the Sun be that diſtant Radiant, and the *Lens* be notably broader than the Image of the Sun, in the Place of the Image a Burning will be excited; and that more vehement than from a concave *Speculum*, if the Image of the Sun be equally diſtant from a *Lens* and *Speculum* equally broad, becauſe of the greater Loſs of Rays at this than at that. If a lucid Body be put in the forementioned Place, the Image of it will be caſt at a very great Diſtance, and will enlighten thoſe Parts that are vaſtly remote.

If the Radiant be not vaſtly diſtant as before, but yet more remote from the *Lens*, than the Place of the Image of a vaſtly diſtant Radiant; beſides the Appearances juſt explained, namely, that the Image will be inverted, *&c.* if the Radiant approaches towards the *Lens*, the Image will recede, and *vice verſa*; till the Radiant comes to the Place of the Image of a very diſtant Radiant, and then its Image will become vaſtly diſtant. All which may be ſeen in a darkened Chamber, receiving no Light but through a convex *Lens*, upon which Radiants at different Diſtances caſt their Rays. The Place of the Image of any Radiant is known, by its being painted moſt diſtinctly upon a white unpoliſhed Plane, ſituated in the Chamber. Nor is there need of ſubjoining any thing more of this Experiment, which is now very common; or of that other, founded upon the ſame optical Principles, in which, by the pellucid Colours of a Picture painted upon Glaſs, and tranſmitting the cloſe Rays of a Flame, an Image is ſhewn painted upon a white unpoliſhed Plane.

If the Radiant be nearer the convex *Lens* than the Image of a vaſtly diſtant Radiant, then its Image will be formed not on the oppoſite, but on the ſame Side of the *Lens*, and its Place according to what has gone before, from the Place of the Radiant being given will be determined. This Image is alwas erect, and greater than the Radiant: And as the Radiant approaches to the *Lens*, the Image likewiſe approaches to it; and in like manner they both recede from it at the ſame time, but the Image more ſwiftly.

Fig. 21. If the convex *Lens* were changed into a concave one, the ſame Conſtruction remains; and by a Calculation (according to the *Corollaries* of *Prop.* XIV.) *e b f*, the Image of a very diſtant Radiant, made by a Plano-concave *Lens* P Q, will be found erect, and on the ſame Side of D with the Radiant, and diſtant from it by the *Diameter* of the Sphere P D Q. Now if the Surface P C Q be alſo ſpherical, and the Radiant from being diſtant becomes near, its Image will be determined by the ſame Conſtruction, if by *Prop.* XV. the Image of the radiant Point placed in the *Axis* of the *Lens* be firſt found. In which Caſe, beſides what has been juſt now ſaid, it is alſo to be obſerved, that the Image approaches to, or recedes from, the *Lens* at the ſame time as the Radiant does, but more ſlowly; as will be plain to any one who will give himſelf the Trouble of a Calculation.

Let the Rays of any Radiant, inflected after ſuch a manner as to be ready to form the Image *e b f* if nothing hindered, be conceived to be intercepted by a Planō-concave *Lens* P Q; upon whoſe concave Surface P D Q, deſcribed on a Diameter equal to the right Line D *b*, they firſt fall: The ſame Conſtruction as before determines the Image made after the Refraction of theſe Rays at the forementioned *Lens*: For (by *Corol.* 4. *Prop.* XV.) the Rays converging towards *b* are ſo refracted from the concave Glaſs, as to become parallel to the *Axis*, that is, they conſtitute a vaſtly diſtant Image; to whoſe extreme Points (by the foregoing *Theor.*) the right Lines E C, F C, or *e* D, *f* D, are directed, and which is conſequently given in Poſition. And in this Caſe the chief Thing to be obſerved is, that, to an Eye placed about B, the diſtant Image appears inverted, in reſpect to the Image that would have been made at *e b f*, without the Intervention of the *Lens*. It will be eaſy to proceed in all other Caſes, according to theſe Examples. *Q. E. F.*

PROP. XX. PROB. XIII.

TO find ſuch a Poſition of the Radiant with reſpect to a given Lens, *that the Image made by the* Lens *may be equal to a given Figure, which is ſimilar to the Radiant: Or, which is the ſame thing, that the Radiant may be to its Image made by the* Lens *in a given Proportion.* Plate II. Fig. 22.

Let a *Lens* of Glaſs be given; for Example; Let A B be the *Semidiameter* of the firſt Surface, and C B the *Semidiameter* of the ſecond Surface. Draw C M at Pleaſure, making any Angle with C A. Let the Proportion of M D to D C be given; namely, that which the homologous Lines of the Radiant and its Image bear to one another. Join A M; to which through B draw B E parallel, meeting the right Line C M in E; M E taken twice will be the ſought Diſtance of the Radiant from the *Lens*. If D *m* be taken in the ſame right Line, but on the other Side of the Point D, equal to the right Line D M, and you join A *m*; B *e*, drawn parallel to this, will cut off another *m e*, which will likewiſe ſatisfy the Problem: For the Radiant placed at the Diſtance of twice *me* from the *Lens*, is to its Image (but then it will be made on the contrary Side of the *Lens* with reſpect to the former Image) as D M to D C.

By *Corol. Prop* XVIII. the radiant Line is to its Image madge by a *Lens*, as their Diſtances from the *Lens*: But if twice M E, or twice *me*, be the Diſtance of the radiant Point from the *Lens*, the Diſtance of the Image of that Point from the *Lens*, will be to the former Diſtance reſpectively, as M D to D C; as will be plain to any one, who will undertake a Calculation according to *Prop*. XV. For if A B be called a, B C b, the Thickneſs of the *Lens* C, M D r, and z the ſought Diſtance of the Radiant from the *Lens* of Glaſs about which Air is circumfuſed; we ſhall have $zz =$

$$\frac{6abz - 2acz \overset{-}{+} 6arz \underset{-}{\times} 2crz \overset{-}{+} 4acr}{3a - c + 3b}$$

If the Thickneſs of the *Lens* be neglected (which is done in the Conſtruction of the Problem) the ſought Diſtance of the Radiant from the *Lens* will be equal to

$$\frac{2ab \overset{-}{\times} 2ar}{a + b}.$$

In the Scheme referred to the *Lens* is made convex on both Sides; but the ſame Conſtruction will ſerve for any *Lens*, ſince from the Variety of *Lenſes* only the Order of the Points A, B, and C is changed. Another Conſtruction of this Problem is eaſily deduced from *Corol. Prop.* XVI.

PROP. XXI. PROB. XIV.

With two given ſpherical Lenſes *or* Specula, *or one* Lens *and one* Speculum, *to make an optical Machine, which, to an Eye ſeeing at a given Diſtance, ſhall diſtinctly repreſent a given near Object under a given Angle, the Diſtance of the Eye from the laſt* Lens *or* Speculum *being likewiſe aſſigned.*

Since the Eye is a Machine made on purpoſe, that the Images of exterior Radiants may be diſtinctly painted upon its Bottom (which is made concave for this End, as *Corol. Theor.* VI. requires) all the reſt of its *Apparatus* conducing only to its Motion or Security, which are neceſſary to be provided for; it is plain, that a given Eye can only ſee diſtinctly at a given Diſtance from the Object. Now if the Eye could place it ſelf at Pleaſure, at ſuch a Diſtance from any Object as is neceſſary to diſtinct Viſion, (all other things, as the Degree of Light, *&c.* being as they ſhould be) Viſion would always be diſtinct. And though there be no Eye ſo ſtiff, as to ſee only at a determinate Diſtance, but can, according to the Flexibility or Mobility of the Parts with which every Eye is endued, apply it ſelf to Objects placed at different Diſtances, and change its Figure according to the Diſtance given; ſo as to be no longer looked upon as the ſame given Eye, but various and mutable, as Occaſion requires: Yet, ſince this Mobility is confined within certain Limits, and there are a great many Objects to which we cannot at Pleaſure come near enough to be within thoſe Limits; 'tis plain, there will be need of an optical Machine to ſee them diſtinctly. But any ſpherical *Lens* or *Speculum* will be ſufficient for this Purpoſe; ſince by its Aſſiſtance the Image of any Radiant (to which we cannot come ſo near as we would) may be brought near us (as is plain from what has been before demonſtrated) and then we ſhall be able to view it, ſince the Eye is ſuppoſed moveable at Pleaſure with reſpect to any thing near.

But becauſe, beſides diſtinct Viſion, our Occaſions ſometimes require us to look into the more minute Parts of an Object, and it is found by Experiment, that an Object ſeen under a leſs Angle than of one Minute, is conſidered by the Obſerver as a Point, and its Parts not at all to be diſtinguiſhed one from another: It often happens, that when the Object is brought nearer the Eye, that the Particles to be obſerved may be ſeen under a ſenſible Angle, and greater than the forementioned one, the Object it ſelf becomes too near the Eye, and is out of the Limits required for diſtinct Viſion. This Inconveniency, if it be the only one, may be remedied by the Aſſiſtance of any given *Lens* or *Speculum*, by *Prop.* X. or XX. where the Image of an Object made by a given *Speculum* or *Lens* is repreſented in any given Meaſure.

But if both the forementioned Inconveniencies urge at the ſame time, they are not to be removed without the Help of two *Lenſes* or *Specula*, or one *Lens* and *Speculum*. Having ſhewn therefore what Aſſiſtance the Sight may receive from a ſingle given *Lens* or *Speculum*, we ſhall proceed to Machines made by two combined together; or to conſtruct the Problem propoſed univerſally. *Plate* II. *Fig.* 23, 24.

Let R be the given Object, S the given Angle in which it is to be repreſented, D the given Diſtance requiſite to diſtinct Viſion, and L the given Diſtance of the Eye from the *Lens*. Make the Triangle A O B, in which the Angle at the *Vertex* O is equal to the given S, and O E perpendicular from the *Vertex* upon the *Baſis* equal to the given D. If we take the middle Point of the Object placed in the *Axis* of the *Lens* or *Speculum*, which in Practice is very convenient, the Triangle A O B muſt be made Iſoſceles. Take O V equal to L. At V place either of the *Lenſes* or *Specula*, having its *Axis* in the right Line O E.

By *Prop.* VI. or XVI. having one *Focus* E of a *Lens* or *Speculum* given, find the other *e*; that is, that the Rays whoſe *Focus* before Incidence is *e*, may have E for their *Focus* after Inflexion at the *Lens* or *Speculum*. Through *e* draw the right Line *a e b* parallel to the right Line A E B, meeting the right Lines V *a*, V *b*, drawn through V parallel to the right Lines V A, V B, in *a* and *b*.

'Tis plain, from *Prop.* IX. or XIX. that if *a e b* be the Radiant, A E B will be its Image: Wherefore if the given near Object R, and another *Lens* or *Speculum*, be placed after ſuch a manner (by *Prop.* X. or XX.) that the Image of the Object R made by this *Lens* or *Speculum*, may obtain the Situation and Magnitude *a e b*; the Microſcope required is made: For *a e b* is the Image of the Object R, the

Image of which Image, ſeen by the Eye placed in O, is A E B. Now this appears under the Angle A O B equal to the given S, and at the Diſtance O E equal to the given D, and conſequently diſtinct; and the Diſtance of the Eye from the laſt *Lens* or *Speculum* in V, is equal to the given L. *Q. E. F.*

If the given Eye be an old Man's, every thing elſe remaining as before, the right Line O E becomes infinite, and the Point *e* is found by *Prop.* III. or XIV.

If the Object propoſed were vaſtly diſtant, the Problem would be impoſſible. But if the Angle S were not given, a Teleſcope from the remaining *Data* might thus be made. In the right Line given in Poſition, tending towards the propoſed diſtant Object, ſtraight forward from O, take O E, O V, equal to D and L: And in V place one of the *Lenſes* or *Specula*, having its *Axis* in V E. By *Prop.* VI. or XVI. one *Focus* E of the *Lens* V being given, find the other *e*; which being ſuppoſed the *Focus* of Rays before Inflexion at V, E may be their *Focus* after Inflexion. In the right Line O E let the other *Lens* or *Speculum*, having its *Axis* in the ſame, be ſo placed, that the Image of the diſtant Object made by it may be ſituated in the right Line *a e b* perpendicular to O E, and the Teleſcope required is made. For the firſt Image of the diſtant Object is in *aeb*: And the Image of this Image is, by Conſtruction, in A E B, whoſe Diſtance from O is equal to the right Line O E or the given D, and conſequently diſtinctly ſeen; and the Diſtance of the Eye from V is equal to the given L. But a diſtant Object being given, by *Prop.* XIX. its Image made by a given *Lens* or *Speculum*, and likewiſe the Image of that Image made by the given V will be given, and conſequently the Angle under which this laſt is ſeen by the Eye in a given Poſition.

But if O be an old Man's Eye, the Angle A O B, becauſe of A B being in this Caſe vaſtly diſtant, is equal to A V B, or *a* V *b*. Therefore the Angles under which a diſtant Object with and without a Machine appears, are as the Diſtances of the *Lenſes* or *Specula* from the common *Focus e*: For ſmall Angles are almoſt as their Cotangles.

SCHOLIUM.

It will be very convenient that the *Lens* or *Speculum*, which immediately receives the Rays of the Object, and forms its firſt Image (and is therefore called the *Object Glaſs*) be as perfect as poſſible: For the Errors or Defects of this *Lens* or *Speculum* affect the Image made by it. And ſince this Image acts the Part of an Object, to be ſeen through the *Speculum* or *Lens* V, (which is neareſt the Eye, and therefore called the *Ocular Glaſs*) its Defects, that is, the Defects or Errors of the Object Glaſs by which it is formed will be greater and more ſenſible, by how much the Image AEB is greater than *a e b*; that is, by how much more perfect (the Object Glaſs remaining the ſame) the Machine is made. But the Errors of the Ocular *Lens* or *Speculum* V, are equally ſenſible, whatſoever be the Image *a e b*, or the Object *Lens* or *Speculum* by which it is produced; that is, the Ocular Glaſs V remaining the ſame, its Defects are equally apparent and diſcoverable, to whatſoever Degree of Perfection the Machine, by changing the Object *Lens* or *Speculum*, which forms the Image *a e b*, be brought: For the Eye being given, the given Ocular *Lens* or *Speculum* V is always at the ſame Diſtance from the Image *a e b*, doing the Office of an Object and its Image A E B, and conſequently ſhews the ſame Defects.

PROP. XXII. PROB. XV.

With three or more given ſpherical Lenſes *or* Specula, *to make a Machine, which to a given Eye ſhall diſtinctly repreſent a propoſed diſtant Object under a given Angle, the Diſtance of the Eye from the laſt* Lens *or* Speculum *being aſſigned.*

By the Help of any one of the given *Lenſes* or *Specula* from the Image of the diſtant Object, and with the two others, by the foregoing *Prop.* make a Microſcope that ſhall repreſent it in the Conditions propoſed, and the Teleſcope required is made.

In like manner with theſe *Lenſes* or *Specula* may a Microſcope be made; and then by adding a fourth, another Teleſcope: In all which we have the Proportion which the Image ſeen by the Eye bears to the Object, or which the Angle under which that is ſeen, bears to the Angle under which this is ſeen without the Machine: And conſequently the Powers of a Machine in promoting Viſion, are, by *Corol. Prop.* IX. and XVIII. eaſily eſtimated.

PROP. XXIII. PROB. XVI.

TO make a ſpherical Lens *of Glaſs, whoſe Thickneſs is given, that ſhall to an Eye ſeeing at a given Diſtance repreſent a given Object placed at a given Diſtance, under a given Angle diſtinctly, the Diſtance of the Eye from the* Lens *being likewiſe aſſigned.* Plate III. Fig. 1.

We have hitherto ſhewn what Aſſiſtance may be had from given *Lenſes* or *Specula*, or both, howſoever combined, in order to ſupply the Defects of Viſion; it remains, that we demonſtrate the Manner of making a *Lens* for given Uſes.

The radiant Line A B, whoſe Diſtance from the *Lens* is V E, is to be repreſented diſtinctly under the viſual Angle *a* O *b*, to an Eye ſeeing diſtinctly at the Diſtance O *e*. In the Iſoſceles Triangle *a* O *b* (whoſe Baſe is *a b*, and Height *e* O) upon the Perpendicular O *e* (ſo ſituated as to biſect the radiant Line A B perpendicularly in E) take O L equal to the given Diſtance of the Eye from the *Lens*. Draw *a* L, *b* L, and produce them to D and F. Make L V equal to the given Thickneſs of the *Lens*. From the right Line V L, given in Poſition, perpendicular to the Glaſs at the Points of Incidence, draw the refracted Rays in Glaſs V α, V β, L H, L G correſponding to thoſe in Air A V, B V, D L, F L. Produce the right Lines H L, G L, till they meet with the right Lines V β, V α in β and α. The right Line α β join'd, will cut the Perpendicular *e* O perpendicularly in ε, becauſe both the Angles of Incidence and of Refraction, on each ſide of the right Line O *e* are equal. From the *Foci* E and ε, and the Vertex V being given, find (by *Corol.* 2. *Prop.* XV.) the Sphere of Glaſs K V M that may refract the Rays in Air diverging from E, ſo as to make them converge towards ε. By the forementioned *Corol.* find alſo N L P, the Surface of a Sphere of Air paſſing through L, that may refract the Rays in Glaſs converging towards their *Focus* ε, ſo as to make them afterwards diverge from *e*: The ſolid Figure M K N P, being made of Glaſs, and terminated by the ſpherical Surfaces K V M, N L P, and a conic Surface whoſe *Axis* is V L, is the *Lens* required. But Care muſt be taken, that the Portions V K, L N be not too great; becauſe all the preceding Demonſtrations hold only true of Rays falling near the *Vertex*.

By Conſtruction, the Rays diverging from E do, after Refraction at the firſt

Surface KVM, converge to the *Focus* ε: Wherefore (by *Prop.* XVII.) the Image of the radiant right Line A B (made by Refraction at K V M) is in the right Line $\alpha\beta$ given in Poſition: But the Ray AV, by Conſtruction, after it is refracted at the Surface KV M comes to the Point α; therefore that Point α is the Image of the Point A, which is produced by the firſt Surface only. In like manner β is the Image of the Point B; and the right Line $\alpha\varepsilon\beta$ is the Image of the radiant right Line A E B. Moreover (by Conſtruction) the Rays within the *Lens* that converge towards ε, after they are refracted at the Surface N L P, diverge from e; wherefore (by *Prop.* XVII.) the Image of the future Image $\alpha\varepsilon\beta$, made by Refraction at the Surface NLP, is in the right Line $a\,e\,b$: But the Ray G L within the *Lens*, proceeding directly towards α, after it is refracted, becomes L F, and proceeds from a; whence the Image of the future Image in α, after Refraction at N L P, becomes a: And ſo likewiſe the Rays within the *Lens* that converge towards β, after Refraction at N L P, diverge from b; that is, aeb is the Image of the Radiant A E B, placed at a given Diſtance V E from the *Lens*, made by the *Lens* N K M P, and to be ſeen diſtinctly by a given Eye in O; becauſe it is at the required Diſtance O*e* from the Eye, and is alſo ſeen under the given Angle *a*O*b* by the Eye O, which is at the given Diſtance O L from the *Lens*, and V L is the given Thickneſs of the *Lens*. *Q. E. F.*

The ſame Conſtruction ſerves, if the Eye of an old Man be given: For in that Caſe *a e b* is at an infinite Diſtance, and the former right Lines *a* L, *b* L muſt be drawn through L parallel to the right Lines O *a*, O *b* given in Poſition, and the Center of the Surface N L P (by *Corol. 6. Prop.* XIV.) is diſtant from L by a third part of the right Line L*e*.

But if the Object propoſed be a diſtant one, the Conſtruction will become much more ſimple: For the Center of the firſt Surface K V M (by *Corol.* 1. *Prop.* XIV.) will be diſtant from the *Vertex* V by a third part of the right Line V*e*.

PROP. XXIV. PROB. XVII.

T*O make a concave ſpherical* Speculum *of Glaſs, whoſe Thickneſs and Diameter of Concavity are given, ſo that the Rays parallel to its* Axis, *reflected from both its Surfaces, ſhall be collected* *in the ſame Point of its* Axis. Plate III. Fig. 2.

Becauſe it is found by Experience, that *Specula* made of Metals neither receive a due Figure and Poliſhing ſo eaſily, nor preſerve it ſo long, it will be convenient to uſe thoſe of Glaſs. Let B then be the *Vertex* of the concave Glaſs Surface E B F, A its Center, and B D the Thickneſs of the *Lens*, be given; we are to find D C the *Semidiameter* of the Surface G D H, ſo that the Rays parallel to A B, and falling upon the concave *Speculum*, as well thoſe that are reflected from the Surface E B F, as thoſe that are twice refracted at the forementioned Surface E B F at their Entrance into it and Emerſion out of it, but reflected from the concave Surface G D H, may all meet in the ſame Point of the *Axis* A B: Or, which is the ſame thing, a *Speculum* of Glaſs is to be made after ſuch a manner, that the two Images of a diſtant Object, made by the two Surfaces of the *Speculum*, may coincide, and conſequently ſo as to be moſt powerful in burning, or in forming the Images of diſtant Objects. *Plate* III. *Fig.* 3.

From any Point L of the infinite right Line S M, on one Side take L N equal to B D, B D, and on the other Side L R equal to twice B D; from N likewiſe directly forward take N T equal to thrice A B, and T V equal to twice N T, and V M alſo equal to twice R L. On the Center R, with the *Radius* R T, deſcribe a Circle meeting the Perpendicular erected at L in K. Join M K; and at K erect the Perpendicular K S meeting the infinite right Line firſt drawn, in S, *Fig.* 2. In the right Line D A, from D towards A, take D C equal to the right Line L S; and on the Center C with the *Radius* C D deſcribe an Arch of a Circle D G, ſimilar to the Arch B E deſcribed on the Center A, and join E G. The ſolid Figure, generated by the Rotation of the plane Figure B D G E about the fixed right Line B D, and conſiſting of Glaſs, is the *Speculum* required. *Q. E. F.*

Fig. 4. But if A B be ſufficiently large in reſpect to the Thickneſs B D, the Problem will admit of this more expeditious Conſtruction. Divide B D in O, ſo that the greateſt Segment B O may be to the leſſer O D, as 5 to 4. Produce B A to C, ſo that C A may be equal to O D; and C will be very nearly the Center of the outward Surface G D H. *Q. E. I.*

We ſhall omit the Demonſtration of theſe two Conſtructions; becauſe it is very eaſily deduced from the following analytical Calculation founded upon *Prop.* V. XIV. and XV. For if $AB = a$, and $BD = c$, we ſhall have

$$DC = \frac{9aa + 18ac + 5cc}{9a + 5c}.$$

In like manner may a *Speculum* be made, if the Radiant be in any other Poſition.

I had determined to have ſubjoined a general Calculation for finding the *Foci* of any *Speculum* or *Lens* univerſally: But that is abundantly done already for *Lenſes* by that excellent Analyſt EDM. HALLEY, in the *Philoſophical Tranſactions* for *November*, 1693, and elegantly applied to particular Caſes.

SCHOLIUM.

Hitherto we have ſhewn what Advantages may be expected from ſpherical *Lenſes* or *Specula*, towards the Conſtruction of Machines: But the different Refrangibility of the Rays of Light, and that in given Rays given, immutable, and annexed to certain Colours, diſcovered by that admirable Philoſopher Sir

ISAAC NEWTON, has ſo much diſturbed our Dioptrical Reaſonings, that no Exactneſs can now be hoped for from *Lenſes*, even though formed into what conoidical Figures we pleaſe. But ſince the Law of *Catoptrics* concerning the Equality of the Angles of Incidence and Reflexion, is preſerved inviolable in Rays, however heterogeneous, as the ſame Great Man obſerves; it is better to uſe a *Speculum* inſtead of the Object *Lens*, which forming the Image of a diſtant Object at a conſiderable Diſtance, diſcovers the Errors that ariſe from the different Refrangibility of Rays ſenſible enough, and not at all to be diſſembled,* if the Rays falling obliquely be admitted by an Aperture ſufficiently large, which is very often neceſſary: But in ſmaller *Lenſes*, ſuch as the ocular ones, the Error is ſo ſmall and inſenſible, that they may be ſtill ſafely uſed.

James Gregory † was the firſt who gave a Specimen of this Sort of *Cata-Dioptrical Teleſcopes*, conſiſting of *Lenſes* and *Specula*, in *Optic. Promot. Prop.* 59. which was many Years afterwards given out by Mr. *Caſſegrain*, a *Frenchman*, for his own. The ſame, upon Phyſical as well as Geometrical Accounts, altered and improved, is publiſhed by Sir ISAAC NEWTON, in his admirable *Theory of* Lights *and* Colours.

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* See the *Latin*.

† Our Author's Uncle.

Since *Specula*, being opake Bodies, cannot have the ſame *Axis* with *Lenſes* without being perforated at their *Vertex*, and conſequently ſuffering an irreparable Loſs (ariſing from both theſe Cauſes) of thoſe Rays that fall near the *Vertex*, and are moſt accurately reflected: The Poſition of the *Axis* may (by *Corol.* 1. *Prop.* II.) be altered as we pleaſe, by the Help of a plane *Speculum*; and by this means (beſides other Advantages) the Neceſſity of Perforation is quite taken away, and the Loſs of Rays falling near the *Axis*, occaſioned by the Opacity, is very much diminiſhed by the Obliquity of the ſecond *Speculum*, which is obſerved by the accurate *Newton* in his *Cata-Dioptrical* Teleſcope. But if, by reaſon of Phyſical Difficulties, in turning and poliſhing proper *Specula*, we muſt ſtill continue the Uſe of *Lenſes*, perhaps it would be of Service to make the Object *Lens* of a different *Medium*, as we ſee done in the Fabrick of the Eye; where the cryſtalline Humour (whoſe Power of refracting the Rays of Light differs very little from that of Glaſs) is by Nature, who never does any thing in vain, joined with the aqueous and vitreous Humours (not differing from Water as to their Power of Refraction) in order that the Image may be painted as diſtinct as poſſible upon the Bottom of the Eye. There are likewiſe other Advantages of the forementioned Artifice in the animal Eye, which belong not to this Place.

A
SUPPLEMENT
To the foregoing Work.

IN order to remove whatever Difficulty might lie in the Way of a Reader who is to begin with this Subject, and interrupt his Progreſs thro' this moſt excellent Treatiſe (beſides having all along in the Tranſlation explained ſuch Paſſages as the Author's Laconic Style has made ſomething too intricate for a Beginner, and corrected ſeveral conſiderable Faults of the Preſs, which had eſcaped his Care) I have thought it proper to ſubjoin a few things, which may perhaps be of Service to obviate ſome farther Difficulties, and ſupply ſome ſeeming Defects.

I. From

I. From Prop. VI. Prob. V.

We may give a Solution of the following Catoptrical Problem, *for finding the* Focus *of any given* Speculum *univerſally; which the Author tells us, at the End of his Book, it was once his Deſign to have done.*

The Problem is this. *The* Focus, *or Point in the* Axis *of a given* Speculum, *from whence, or towards which, Rays proceed, being given, to find the* Focus *or Point where thoſe Rays will meet again, after they fall upon, and are reflected by, a given* Speculum.

Now this Problem being ſolved in one Caſe, namely, that of Rays falling upon a *convex Speculum*, and diverging from a certain Point in the *Axis* of the *Speculum*, will, *mutatis mutandis*, be applicable to all other Caſes whatever, whether the *Speculum* be *convex*, *concave*, or *plane*, and whether the Rays fall *diverging*, *converging*, or *parallel*.

[*Plate* I. *Fig.* 9.] Let then B D be a given *convex Speculum*, whoſe Center is A, and E a given Point in its *Axis*, from whence the Rays which fall upon the *Speculum* diverge: It is required to find the *Focus*, or Point C, in which the Rays E D diverging from the *Focus*, or Point E, do, after Reflexion from the *convex Speculum* BD, meet. Call E B the given Diſtance of the Point E from B the *Vertex* of the *Speculum*

lum d, A B the given *Radius* of the *Speculum r*, and B C the Diſtance of C the *Focus* required from B the *Vertex* of the *Speculum*, *x*. By *Prop.* VI. *Prob.* V. A C $(r-x)$: C B (x) : : A E $(d+r)$: E B (d). From whence ariſes the Equation, $dr-dx=dx+rx$, which gives us B C $(x)=\frac{dr}{2d+r}$ for our general Rule; which ſhews, that in the Caſe of *diverging Rays* falling upon a *convex Speculum*, the *Focus* C is always affirmative, and to be taken from the *Vertex* B directly forward: And that the greater *d* is, the greater will be the *focal* Diſtance, till at laſt *d* becoming infinite, and conſequently the finite Term $+r$ vaniſhing, it will be $x=\frac{dr}{2d}=\frac{r}{2}$ or half the *Radius*; that is, the *Radiant* receding from a *convex Speculum*, the *Image* will alſo recede beyond the *Vertex* of the *Speculum*; but ſo ſlowly, that when the Radiant becomes vaſtly diſtant, the Image will be got no farther than the middle Point, between the *Vertex* and Center of the *Speculum*: And *vice verſa*, the nearer the Radiant approaches to the *Speculum*, the nearer the Image approaches to it, till at laſt they both meet and coincide at the *Vertex*. According to *Obſervat*. 2. *Schol. Prop.* VIII.

If E B (d) be equal to A B (r), the *focal* Diſtance B C becomes $= \frac{r}{3}$: But if E B (d) be equal to half A B ($\frac{1}{2}r$) the *focal* Diſtance will in this Caſe become $= \frac{r}{4}$.

Now to apply this univerſal Canon to all other poſſible Caſes; the Terms of which it conſiſts remaining always the ſame, 'tis only changing the Signs + or − according as the Caſe requires: For in the Caſe of *converging Rays*, the Point in the *Axis* of the *Speculum* towards which they converge, is on the other Side of the *Vertex* of the *Speculum*, and conſequently its given Diſtance, in reſpect to the former, is *negative*, or $-d$. For the ſame Reaſon, in the Caſe of a *concave Speculum*, the Center of the ſpherical Surface lying on the contrary Side of the *Vertex*, the given *Radius* becomes *negative*, or $-r$. And in theſe Caſes reſpectively, whereſoever the Quantity d or r occurs, it muſt have a contrary Sign to what it had before.

If then *converging Rays* fall upon a *convex Speculum*, d being in this Caſe negative, the Rule will be $\frac{-dr}{-2d+r}$; which ſhews, that when $2d$ exceeds r, the *Focus* will be ſtill affirmative; but if $2d$ be leſs than r, the *Focus* will be negative, or on the contrary Side of the *Vertex* of the *Speculum*: That is, if the Point behind the *Speculum*, towards which the Rays converge, be at a greater Diſtance from the *Vertex* of the *Speculum* than half the *Radius*, the *Focus* is ſtill to be taken from B the *Vertex* directly forward, according to *Corol. Prop.* IV. But if its Diſtance be leſs than half the *Radius*, then the *Focus* muſt be taken from the *Vertex* B backward, according to *Corol. Prop.* VI. When d is equal to $\frac{1}{2}r$, the *focal* Diſtance becomes $= \frac{-dr}{0}$, and is conſequently infinite: That is, Rays converging to a Point in the *Axis* of a convex *Speculum*, at an equal Diſtance between the Center and *Vertex*, will, after Reflexion, proceed parallel, according to *Corol. Prop.* III. When d is equal to r, the *focal* Diſtance becomes $= r$; that is, Rays converging towards the Center of the *Speculum*, are reflected by a convex *Speculum* back again upon themſelves.

If *parallel Rays* fall upon a *convex Speculum*, d in this Caſe becoming infinite, the Rule will be $\frac{dr}{2d} = \frac{1}{2}r$; which ſhews, that

Rays falling parallel upon a convex *Speculum*, are collected in a Point at the Diſtance of half the *Radius* behind the *Speculum*, according to what has been demonſtrated at *Prop.* III. And conſequently a vaſtly diſtant Radiant will have its Image formed in this Point; whence the Sun's Beams will be there collected, and heat or burn any thing placed therein.

If *diverging Rays* fall upon a *concave Speculum*, the *Radius* being in this Caſe $-r$, the Rule will be $\frac{-dr}{2d-r}$; which ſhews that when d is leſs than $\frac{1}{2}r$, the *Focus* is *affirmative*; when d is equal to $\frac{1}{2}r$, the *Focus* is *infinite*; and when d is greater than $\frac{1}{2}r$, the *Focus* is *negative*; and when d is equal to r, the *focal* Diſtance is $=-r$: That is, if the Point in the *Axis* of the *Speculum* from which the Rays diverge be nearer the *Vertex* than half the *Radius*, the *Focus* will ſtill be behind the *Speculum*, according to *Prop.* V. If it be juſt at the Diſtance of half the *Radius*, the Rays after Reflexion will proceed parallel, according to *Prop.* III. If it be at a greater Diſtance than half the *Radius*, the *Focus* after Reflexion will be on the ſame Side of the *Speculum* with the *Focus* before Reflexion, or Point from which the Rays diverge. If it be at the Diſtance of the whole *Radius*, the Rays after Reflexion meet in the ſame Point from which they firſt diverge. It is moreover manifeſt, that the more *d* exceeds ½ *r*, the leſs will be the *negative focal* Diſtance; but if *d* be infinite, the *focal* Diſtance in this Caſe $= \frac{-dr}{2d} = \frac{1}{2}r$, can be no leſs than half the *Radius*: And on the contrary, the leſs *d* exceeds ½ *r*, the greater will be the *focal* Diſtance; till at laſt, *d* becoming equal to ½ *r*, the *focal* Diſtance in this Caſe $= \frac{-dr}{0}$ becomes infinite. But when *d* grows any thing leſs than ½*r*, the *Focus* becoming *aſſirmative*, is thrown at a greater Diſtance on the contrary Side of the *Speculum*; and by how much *d* is leſs than ½*r*, the leſs will be this affirmative *focal* Diſtance. So that a *Radiant* placed at a greater Diſtance than half the *Radius* from a *concave Speculum*, the farther it recedes from the *Speculum*, the nearer its *Image* which is on the ſame Side approaches to the *Speculum*, and at the Center of the *Speculum* they meet, and afterwards croſs one another, till the Radiant becoming *vaſtly diſtant*, the Image will come within half the Diſtance of the *Radius* from the *Speculum*: And *vice verſa*, the nearer the Radiant approaches the *Speculum*, the farther the Image recedes from it, and at the Center of the *Speculum* both meet, and afterwards croſs one another, till at laſt the Radiant coming to half the Diſtance of the *Radius* from the *Speculum*, the Image becomes *vaſtly diſtant:* Whence if a lucid Body be placed at the Diſtance of half the *Radius* from a concave *Speculum*, it will enlighten Places that are vaſtly diſtant. If the Radiant comes nearer the *Speculum* than half the *Radius*, the Image will be caſt from before the *Speculum* to a great Diſtance on the contrary Side; and the nearer the Radiant now approaches to the *Speculum*, the nearer will the Image likewiſe approach to it, till at laſt they both coincide at its *Vertex*, and *vice verſa*, according to *Obſervat.* 1. *Schol. Prop.* VIII.

If *converging Rays* fall upon a *concave Speculum*, d and r being in this Caſe both negative, the Rule will be $\frac{dr}{-2d-r}$; which ſhews that the *Focus* is always *negative:* That is, Rays that fall converging upon a concave *Speculum*, will always be collected in a Point or *Focus* on this Side the *Speculum*, according to *Corol. Prop.* V. If d be equal to r, the *focal* Diſtance is $=\frac{-r}{3}$; but if d be equal to $\frac{1}{2}r$, the *focal* Diſtance is $=\frac{-r}{4}$.

If *parallel Rays* fall upon a *concave Speculum*, *d* in this Caſe becoming infinite, the Rule will be $\frac{-dr}{2d} = -\frac{1}{2}r$: That is, Rays falling parallel upon a concave *Speculum*, are collected in a Point at the Diſtance of half the *Radius* on this Side the *Speculum*, as has been demonſtrated at *Prop.* III. Whence if the Sun's Beams be received upon ſuch a *Speculum*, the ſame will be the burning Point.

If *diverging Rays* fall upon a *plane Speculum*, the *Radius r* being infinite, the Rule will be $\frac{dr}{r} = d$; that is, the *Focus* of diverging Rays reflected by a plane *Speculum*, will be at as great a Diſtance behind the *Speculum*, as the Point from which they diverge is before the *Speculum*, according to *Prop.* II. And conſequently the Image of any Radiant, made by Reflexion from a plane *Speculum*, will be ſeen as far behind the *Speculum* as the Radiant is before the *Speculum*; and they will both not only recede from and approach to the *Speculum* at the ſame time, but likewiſe keep an equal Pace one with another.

If *converging Rays* fall upon a *plane Speculum*, *d* being negative, and *r* infinite, the Rule will be $\frac{-dr}{r} = -d$; that is, Rays converging to a Point at a certain Diſtance behind a plane *Speculum*, will have their *Focus* at an equal Diſtance from the *Speculum*; according to *Corol.* 1. *Prop.* II.

If *parallel Rays* fall upon a *plane Speculum*, both d and r being in this Caſe infinite, the *focal* Diſtance $\frac{dr}{2d+r}$ will be alſo infinite: That is, Rays falling parallel upon a plane *Speculum*, will be reflected back parallel.

It is worth obſerving, that the Conſideration of *diverging Rays* relates to Objects that are near us, and ſuch as we examine with our naked Eye, or by the Help of a Microſcope: *Parallel Rays* are conſidered when we have to do with Objects vaſtly diſtant, and ſuch as we look at through Teleſcopes. And *converging Rays* fall under our Conſideration, when the Rays proceeding diverging or parallel from any Object are by one *Speculum* or *Lens* made to converge, and then intercepted by the Interpoſition of another *Speculum* or *Lens*, before they arrive at their Point of Convergence; which is of great Uſe in examining the Effects of optical Machines, made by a Combination of more than one *Lens* or *Speculum*, and conſtructing ſuch as are proper for any aſſigned Purpoſe, where this Contrivance is often abſolutely neceſſary.

There are two farther *Uſes* of this Method; the firſt is, to determine what Degree of *Convexity* or *Concavity* is neceſſary for a *Speculum* to repreſent a given Object at a given *Focus*. And this is very eaſily done from the Equation firſt found $dr - dx = dx + rx$: For d and x being given, the *Radius* of Convexity $r = \frac{2dx}{d-x}$. Where it is plain, that if x be greater than d, r will be a negative Quantity, and the Problem impoſſible for a convex *Speculum*; That is, if it be required to repreſent a given Object, at ſuch a given *Focus*, whoſe Diſtance on the other Side of the *Speculum* ſhall be greater than the Diſtance of the Object on this Side; inſtead of a convex Glaſs, we muſt uſe a concave, whoſe negative *Radius* will be $= \frac{2dx}{x-d}$. After the ſame manner the Degree of Concavity is found from the ſame Equation, only changing the Sign of r from + to —: For d and x being given, we ſhall have the *Radius* of Concavity $r = \frac{2dx}{x-d}$. Where it is manifeſt, that if d be greater tan x, r will be a negative Quantity, and the Problem impoſſible for a concave *Speculum*: That is, if it be required to repreſent a given Object, at ſuch a given *Focus*, whoſe Diſtance on the other Side of the *Speculum* ſhall be leſs than the Diſtance of the Object on this Side, inſtead of a concave *Speculum*, we muſt make uſe of a convex, whoſe affirmative *Radius* is $= \frac{2dx}{d-x}$. And in both Caſes if d be equal to x, then the *Radius* either of Convexity or Concavity $= \frac{2dx}{\substack{+d-x \\ -\;\;\;+}} = \frac{2dx}{0}$ will be infinite, and the Problem will be impoſſible for either a convex or concave *Speculum*: That is, if it be required that a given Object ſhall be repreſented at ſuch a given *Focus*, whoſe Diſtance on the other Side of the *Speculum* ſhall be equal to the Diſtance of the Object on this Side, inſtead of a *convex* or *concave* *Speculum*, a *plane Speculum* is the only one that can be uſed. If d be infinite, the *Radius* of Convexity becomes $\frac{2dx}{d} = 2x$; but the *Radius* of Concavity $\frac{2dx}{-d} = -2x$, will have a negative Value: Which ſhews, that the Problem is impoſſible in the Caſe of a concave *Speculum*; that is, a vaſtly

diſtant

diſtant Object cannot be repreſented at any given *Focus* behind a concave Speculum; but may by a Convex, whoſe Radius of Convexity muſt be equal to twice the given *focal* Diſtance behind the Speculum. If x be infinite, then the Radius of Convexity $= \frac{2dx}{-x} = -2d$ has a negative Value, and the Problem is impoſſible for a convex Speculum; but the Radius of Concavity is $= \frac{2dx}{x} = 2d$. Whence if we would have a given Object repreſented at an infinite Diſtance behind the Speculum, we can make uſe of none but a concave Speculum, whoſe Radius of Concavity muſt be equal to twice the given Diſtance of the Object.

Hitherto we have conſidered the *Focus* as affirmative, that is, behind or beyond the Speculum; but the ſame Rule, only changing the Sign of x in the Equation $x = \frac{dr}{2d+r}$, will equally hold if we would have the *Focus* negative, or on the ſame Side of the Speculum with the Object: For in the Caſe of a convex Speculum we ſhall have the Radius of Convexity $= \frac{-2dx}{d+x}$ always of a negative Value, and conſequently the Problem is always impoſſible for a convex Speculum; but in the Caſe of a concave Speculum, the Radius of Concavity will be $= \frac{-2dx}{-d-x}$: Which ſhews, that the Problem is always poſſible for a concave Speculum, be the Quantities d and x as they will. If we would have $d=x$, then the Radius of Concavity will be alſo equal to d or x: That is, a concave Speculum, whoſe Radius is equal to the Diſtance of the Object from the Speculum, will reflect the Image into the ſame Place with the Object.

The other *Uſe* of this Method is, from the *Image* and *Speculum* given, to find the *Diſtance* of the Object from the *Speculum*: That is, in the Equation firſt found $dr - dx = dx + rx$, x and r being given, we are to find d, which will conſequently be $= \frac{rx}{r-2x}$ in *convex Specula*, and in *concave* $= \frac{rx}{r+2x}$. Whence 'tis plain, that in *convex Specula* the Problem will be impoſſible when x exceeds $\frac{1}{2}r$, but in *concave Specula* it will always be poſſible: That is, if the Image is to be at a greater Diſtance than $\frac{1}{2}$ the Radius behind the Speculum, it cannot be made by an Object placed before a convex Speculum at what Diſtance ſoever: But let the Diſtance of the Image behind the Speculum be what it will, it may be formed by an Object expoſed at ſome certain Diſtance before a concave Speculum. If the given *Focus* be negative, or the Image on the ſame Side of the Speculum with the Object, then changing the Sign of *x*, in the forementioned Equation, we ſhall have, in the Caſe of *convex Specula*, $d = \frac{-rx}{r+2x}$; and in the Caſe of *concave Specula*, $d = \frac{-rx}{r-2x}$. So that the Problem will always be impoſſible for *convex Specula*, and only poſſible for *concave* ones, when $\frac{1}{2}r$ does not exceed *x*, or when the Image is not nearer the Speculum than by half the Radius.

II. From Corol. Prop. IX.

WE *may deduce a Solution of the following* Catoptrical Problem, *of magnifying or diminiſhing a given Object by a given* Speculum *in any aſſigned Proportion.*

The Problem is this: *To find at what Diſtance from a given* Speculum *it is ne-*

ceſſary to place an Object, in order that the homologous Lines of the Image made by the Speculum *may bear any aſſigned Proportion to thoſe of the Object.*

Since it is evident from this *Corol.* that the homologous Lines of the Radiant and Image are to one another as their Diſtances from the Speculum reſpectively: It follows, that if *b* to *c* expreſs the Proportion which the homologous Lines of the Object and Image are to bear to one another, *b* will be to *c*, as *d*, the Diſtance of the Object, to *x*, the *focal* Diſtance of the Image. Whence, if we compare this Value of *x* with that delivered in the foregoing Problem, we ſhall have $\frac{cd}{b} = \frac{dr}{2d+r}$: And conſequently the Diſtance required $d = \frac{br - cr}{2c}$ will be our general Rule; and will, *mutatis mutandis*, extend it ſelf to all poſſible Caſes whatever, though in its preſent Form it regards the Caſe of *convex Specula* in particular: For *concave Specula* 'twill ſtand thus $d = \frac{cr - br}{2c}$. If the Image be deſired on the ſame Side of the Speculum with the Object, *x* being negative, in the Caſe of *convex Specula*, 'twill be $d = \frac{-br-cr}{2c}$; and in the Caſe of concave $d = \frac{br+cr}{2c}$.

From whence 'tis plain, that there is no magnifying an Object by a concave Speculum; for c being in this Caſe greater than b, the Rule for the affirmative *Focus* $d = \frac{br-cr}{2c}$ will have a negative Value, and that for the negative *Focus* $d = \frac{-br-cr}{2c}$ has always a negative Value: So that we can only diminiſh an Object, and make it appear leſs, by a convex Speculum, and that only when the *Focus* is affirmative, or the Image to be repreſented behind the Speculum. And by a concave Speculum there is no diminiſhing an Object, as long as the *Focus* is affirmative; for b being greater than c, the Rule in that Caſe $d = \frac{cr-br}{2c}$ will have a negative Value: So that we can only magnify an Object, and make it appear greater behind a concave Speculum. But if the *Focus* be negative, and the Image and Object to be both on the ſame Side of the Speculum, the Rule being $d = \frac{br + cr}{2c}$, ſhews that in this Caſe a concave Speculum will magnify or diminiſh an Object in what Proportion we pleaſe.

It is to be obſerved, that if the Object be a right Line, the Proportion *b* to *c* will expreſs the Proportion between the Object and Image themſelves; but if the Object be a plane Figure, the Proportion *b* to *c* will be only ſubduplicate of that which the Object bears to the Image, as we learn from *Euclid*. So that if *b* to *c* be as 2 to 1, the Object and Image will be as the Squares of theſe Numbers, or as 4 to 1. But it muſt alſo be noted, that Painters uſually meaſure the Largeneſs of their Figures by the ſimple Proportion of their homologous Lines; ſo that when they ſpeak of an human Figure twice as big as the Life, their Meaning is, that the homologous Lines of this Figure are twice as great as thoſe of the Life; or that the Dimenſions of every Member in Length and Breadth are twice as large as thoſe of the Men repreſented; though properly ſpeaking, the Picture is four times as big as the Life.

If the Out-Lines of the Image be deſired twice as big as the Life, and the *Focus* affirmative; *c* being in this Caſe greater than

b, the

b, the Problem will be impoſſible for any convex Speculum; but the Object being placed before a concave Speculum at the Diſtance $\frac{cr - br}{2c} = \frac{r}{4}$, or $\frac{1}{4}$ of the Radius, will have its Image magnified in the Proportion aſſigned. If we would have the *Focus* negative, and the Image repreſented on the ſame Side of the Speculum with the Object, ſtill the Problem will be impoſſible for a convex Speculum; but if the Object be placed before a concave Speculum at the Diſtance $\frac{br + cr}{2c} = \frac{3r}{4}$, or $\frac{3}{4}$ of the Radius, its Image made by the Speculum will be magnified in the Proportion aſſigned.

Thus let the Proportion b to c be what it will, the Rule will always give us the Diſtance, at which the Object muſt be placed before the given Speculum, in order to have its Image magnified or diminiſhed in that Proportion. I ſhall only add one Inſtance more; and that is, ſuppoſing b and c were equal, and the *Focus* affirmative: In this Caſe we ſhall have both for convex and concave *Specula* $d = \frac{\pm br \mp cr}{2c} = \frac{0}{2c} = 0$:

That is, the Object muſt be placed at the very Vertex of the Speculum; in which Caſe we know both Object and Image coincide. If the *Focus* were negative, then no convex Speculum will do; and the Rule for *concave Specula* will be $d = \frac{br + cr}{2c} = \frac{2cr}{2c} = r$: That is, the Object muſt be placed in the Center of the Speculum.

There are two farther *Uſes* to be made of this Method; the firſt is, the Diſtance at which the Object is to be placed before the Speculum, and the Proportion in which the Image is to be magnified or diminiſhed, being given, to find what Degree of Convexity or Concavity the Speculum ſhould have, in order to magnify or diminiſh the Image in the Proportion aſſigned: That is, in the Rule $d = \frac{b - cr}{2c}$; d, b, and c being given, we are required to find r; which will give us, in the Caſe of a convex Speculum, and an affirmative *Focus*, $r = \frac{2cd}{b - c}$; and in the Caſe of a concave Speculum, $r = \frac{2cd}{c - b}$. If the *Focus* be negative, for convex *Specula*, the Rule ſtands $r = \frac{2cd}{-b-c}$; for concave, $r = \frac{2cd}{b+c}$.

From all which it appears, that if c be greater than b, or if the Image be deſired greater than the Object, and to be repreſented behind the Speculum, no Convex will do, but a Concave will, whoſe Radius is $\frac{2cd}{c-b}$. As likewiſe, if b be greater than c, or the Image be deſired leſs than the Object, and to be repreſented behind the Speculum, no Concave will do, but a Convex will, whoſe Radius muſt be $\frac{2cd}{b-c}$. If the *Focus* be required negative, or the Image to be on the ſame Side of the Speculum with the Object; the Problem is altogether impoſſible for a convex Speculum, whether to magnify or diminiſh; and always poſſible for a concave Speculum either to magnify or diminiſh.

The other *Uſe* we may make of this Method is, from the Diſtance of the Object before the Speculum, and the Radius of Convexity or Concavity being given, to find the Proportion b to c, which the Object will bear to its Image made by the given Speculum: That is, in the forementioned Rule, having d and r given, to find the Proportion of b to c. Whence in the Caſe of a convex Speculum, and an affirmative *Focus*, 'twill be $b . c :: 2d + rr$; and in the Caſe of a Concave, $b . c :: r - 2d . r$. But if the *Focus* be negative, in *convex Specula*, 'twill be $b . c :: - 2d - r . r$; and in *concave*, $b . c :: 2d - r . r$.

So then in the Caſe of a convex Speculum, and the Image behind the Speculum, b will always be greater than c; becauſe $2d + r$ is of neceſſity greater than r: And in the Caſe of a Concave, b will always be leſs than c for the like Reaſon; and if d be equal to $\frac{1}{2} r$, then c will be infinite in reſpect to b. But if the Image be required on the ſame Side of the Speculum with the Object, in *convex Specula*, 'twill be found always impoſſible; and in *concave*, poſſible in all Caſes whatever, both of magnifying and diminiſhing: For if d be greater than r, then b will be greater than c; if d be leſs than r, b will be leſs than c; and if d be equal to $\frac{1}{2} r$, then c will be infinitely greater than b.

III. At

III. At Corol. 3. Prop. XV.

THE *Author gives a Conſtruction to find the* Focus *of Rays refracted at a ſpherical Surface; and towards the latter end of that* Corol. *applies the ſame to the Caſe of a* plane *Surface: Affirming, that in this Caſe the right Line* F M, *which determines the* Focus *by its meeting the* Axis B A *produced ſomewhere in* C, *will be the ſame with a right Line, joining the Point* F *with another Point taken in* B E *produced, at ſuch a Diſtance from* B, *as to make that Diſtance bear the ſame Proportion to* B E *which* A N *does to* A M; *becauſe the Tangents of Angles are reciprocally as their Cotangents.*

[*Plate* III. *Fig.* 5.] That this may be more eaſily conceived; Suppoſe the Surface B D plane, and take the Point C upon B E produced, ſo as that B C may be to B E as A N to A M, and join F C. *We are to ſhew that the right Line* F M *produced, will in this Caſe meet the* Axis A B *produced, in the Point* C, *and there determine the* Focus. Join E N, E M; and on the Center E with any Radius, as E A, deſcribe an Arch of a Circle A G, and draw the Tangents of the Angles A E N, A E M, which will alway be as A N to A M, and are in the preſent Caſe thoſe very Lines, as likewiſe their reſpective Cotangents G H, G K. Since A, the Center of the plane Surface B D, and conſequently the right Line A M N, is at an infinite Diſtance from B, the right Line F M becomes parallel to E M; and conſequently if produced beyond F, will meet the Axis A B produced ſomewhere, ſuppoſe at C, ſo as to make the Triangle C B F ſimilar to the Triangle K G E. And therefore the right Line E H, paſſing thro' the Vertex of both thoſe Triangles, will cut their Baſes C B, G K ſimilarly in E and H, ſo as to make B C to B E, as G K to G H. But becauſe the Tangents of Angles are reciprocally as their Cotangents, A N is to A M, as G K to G H, therefore B C is to B E, as A N to A M. And conſequently the Point C, where the right Line F M produced meets the Axis A B produced, is that very Point C taken at firſt upon B E produced, ſo as that B C may be to B E, as A N to A M. *Q. E. D.*

IV. At Prop. XVI. Prob XI.

THE Author recommends the Uſe of analytical Calculations, for finding the Foci *of* Lenſes, *as far better than the very niceſt* geometrical Conſtructions: *For which*

which Reaſon it cannot be improper to give the leſs skilful Reader an Example, that he may ſee how ſuch Calculations *are to be managed.*

[*Plate* III. *Fig.* 6.] Suppoſe B D to be the given *Lens*, and E a Point in its Axis, from which Rays diverging fall upon the *Lens*, A the Center of its firſt ſpherical Surface, and C the Center of its ſecond ſpherical Surface, B D the Thickneſs of the *Lens*, and I to R the *Ratio* of Incidence to Refraction. And it is required to find F the *Focus* of thoſe Rays after Refraction at both Surfaces of the *Lens*. We muſt firſt find f the *Focus* of thoſe Rays after their Refraction at the firſt Surface only, or their *Focus* of *Tranſition*. Call E B d, B D t, A B r, C D ϱ, B f x, D F y. By *Propoſit.* XV. $\mathrm{EA}\,(d+r) : \mathrm{A}f\,(x-r) + f\mathrm{B}\,(x) : \mathrm{BE}\,(d) :: \mathrm{I} : \mathrm{R}$: Whence multiplying the Extremes and Means, $\mathrm{R}dx + \mathrm{R}rx = \mathrm{I}dx - \mathrm{I}dr$; and $\mathrm{B}f\,(x) = \dfrac{\mathrm{I}dr}{\mathrm{I}d - \mathrm{R}d - \mathrm{R}r}$. Whence it is plain, that if d be ſo great in reſpect to r, that $\mathrm{I}d$ exceeds $\mathrm{R}d + \mathrm{R}r$, the *Focus* f is affirmative, and to be taken from B the Vertex of the refracting Surface directly forward, as at *Fig.* 6: If $\mathrm{I}d$ is leſs than $\mathrm{R}d + \mathrm{R}r$, the *Focus* f is negative, and to be taken from B backwards, as at

Fig. 7; and if Id be equal to $Rd+Rr$, Bf becomes infinite, and the Rays proceed parallel.

Thus having found f the *Focus*, after the firſt Refraction, we may, by the ſame means, find F the *Focus* after the ſecond Refraction. For, by the ſame *Prop.* XV. $fC\left(\frac{Idr}{Id-Rd-Rr}-t+\varrho=\frac{Idr-Idt+Rdt+Rrt+Id\varrho-Rd\varrho-Rr\varrho}{Id-Rd-Rr}\right):$ $CF\,(y+\varrho)+FD\,(y):Df\left(\frac{Idr}{Id-Rd-Rr}-t=\frac{Idr-Idt+Rdt+Rrt}{Id-Rd-Rr}\right)::$ $R:I:$ (I to R at the Emerſion of Rays from any Lens, being as R was to I at their Immerſion into it.) Wherefore multiplying the Extremes and Means, we have $I^2dry-I^2dty+IRdty+IRrty+I^2d\varrho y-IRd\varrho y-IRr\varrho y=IRdry-IRdty+R^2dty+R^2rty+IRdr\varrho-IRdt\varrho+R^2dt\varrho+R^2rt\varrho$: And conſequently, $DF\,(y)=$

$$\frac{IRdr\varrho-IRdt\varrho+R^2dt\varrho+R^2rt\varrho}{I^2dr-IRdr-I^2dt-+2IRdt-R^2dt+IRrt-R^2rt+I^2d\varrho-IRd\varrho-IRr\varrho}.$$

Which Equation, if we put $p=\frac{R}{I-R}$, may be abridged, and reduced to $DF\,(y)=$

$$\frac{Ipdr\varrho-Rd\varrho t+Rpr\varrho t}{Idr-Idt+Rrt+Id\varrho+Rdt-Ipr\varrho}.$$

And it is evidently the ſame with that, which the famous Dr. *Halley* has given long ago, in the *Philoſophical Tranſactions*, *for finding the* Foci *of optical Glaſſes univerſally.*

This Calculation being general, will ſerve for all Sorts of *Lenſes*, be the Matter of which they are made, and the ambient *Medium* what they will, or whatever be the *Ratio* of I to R: And tho' it is made for *Lenſes* whoſe Surfaces are both *convex*, yet, *mutatis mutandis*, it will extend to *Lenſes* of any other Figure whatever, whether *double-convex* or *double-concave*, *plano-convex* or *plano-concave*, or *convexo-concave*, which laſt Sort are commonly called *Meniſci.* For the Radius of a *concave* Surface being on the contrary Side, or *negative* with reſpect to that of a *convex*, and the Radius of a *plane* Surface *infinite*; 'tis only changing all the Signs + or — with which the Radius of the reſpective Surface, which we would have *concave* inſtead of convex, is affected in the general Rule; or making all the Terms infinite, which involve the Radius of the reſpective Surface, which we would have *plane* inſtead of convex. So likewiſe if we would have it extend to other Rays beſides *diverging* ones; the Diſtance of the Point where *converging* Rays meet, from the firſt Surface of the *Lens*, being on the contrary Side or *negative*, in reſpect to that of *diverging* Rays, and the Diſtance of the

Point where *parallel* Rays meet, from the ſame Surface, being *infinite:* 'Tis only changing the Signs of all the Terms where we meet with *d*, if the Rays are ſuppoſed *converging*; or making thoſe ſame Terms infinite if the Rays are ſuppoſed *parallel*.

In the Caſe of a *double Convex* of Glaſs, if the ambient *Medium* be Air, I being to R as 3 to 2, we ſhall have the *focal* Diſtance from the ſecond Surface of the *Lens*,

$$y = \frac{6dr\rho - 2d\rho t + 4r\rho t}{3dr - dt + 2rt + 3d\rho - 6r\rho}$$

: If the ambient *Medium* be Water, I being to R as 9 to 8, the Rule will be

$$y = \frac{72dr\rho - 8d\rho t + 64r\rho t}{9dr - dt + 8rt + 9d\rho - 72r\rho}.$$

For a *double Convex* of Water, and the ambient *Medium* of Air, I being to R as 4 to 3, the Rule is

$$y = \frac{12dr\rho - 3d\rho t + 9r\rho t}{4dr - dt + 3rt + 4d\rho - 12r\rho}.$$

And for a *double Convex* of Diamond, in a *Medium* of Air, I being to R as 5 to 2, the Rule would be

$$y = \frac{\frac{10}{3}dr\rho - 2d\rho t + \frac{4}{2}r\rho t}{5dr - 3dt + 2rt + 5d\rho - \frac{10}{3}r\rho}.$$

If the Thickneſs of the *Lens* be neglected, which is generally not conſiderable, the Terms where *t* occurs being rejected, the Rule is ſtill farther abridged to

$y = \frac{pdr\rho}{dr + d\rho - pr\rho}$. Where it is evident, that if d be ſo ſmall in reſpect to r and ρ, that $dr + d\rho$ is leſs than $pr\rho$, the *focal* Diſtance y will be negative, and $= \frac{pdr\rho}{-dr - d\rho + pr\rho}$, or the Rays after the two Refractions at both Surfaces of the *Lens*, will ſtill proceed diverging from ſome Point, before the ſecond Surface of the *Lens*; and if $dr + d\rho$ be equal to $pr\rho$, y is infinite, and the Rays after Emerſion from the *Lens* proceed parallel. The Error in neglecting t is ſo ſmall, that if, for the Eaſe of the Calculation, we ſuppoſe a *Lens* of Glaſs equally *convex* on both Sides, and expoſed to *parallel Rays*, r being in this Caſe equal to ρ, and d infinite, the *focal* Diſtance, when t is neglected, is $\frac{2drr}{2dr} = r$, and when it is conſidered $\frac{6rr - 2rt}{6r - t}$; which is only $\frac{rt}{6r - t}$, or nearly $\frac{1}{6}t$ leſs than the former. In the Caſe of *converging Rays* falling upon a *double Convex* of Glaſs, we have $y = \frac{-2dr\rho}{-dr - d\rho - 2r\rho}$, always affirmative: And if the Rays are *parallel*, d being infinite, 'twill be $y = \frac{2dr\rho}{dr + d\rho} = \frac{2r\rho}{r + \rho}$; which alſo gives the *Focus* always affirmative, or behind the *Lens*. *Diverging Rays* falling upon a *double Concave*, give $y = \frac{2dr\rho}{-dr - d\rho - 2r\rho}$ always negative, as in the Caſe of *converging Rays* on a *double Convex* 'twas always affirmative: But if the Rays are *converging*, it will be $y = \frac{-2dr\rho}{dr + d\rho - 2r\rho}$ affirmative, when $dr + d\rho$ is leſs than $2r\rho$, or when the *Focus* of *diverging Rays* collected by a *double Convex* is negative, and *vice verſa*: If the Rays are *parallel*, 'tis $y = \frac{2dr\rho}{-dr - d\rho} = \frac{2r\rho}{-r - \rho}$, always negative. A *plano-convex* Glaſs, the *plane* Surface being expoſed to *diverging* Rays, gives, r being infinite, $y = \frac{2dr\rho}{dr - 2r\rho} = \frac{2d\rho}{d - 2\rho}$, affirmative or negative, according as d is greater or leſs than 2ρ; if expoſed to *converging* Rays, $y = \frac{-2d\rho}{-d - 2\rho}$, always affirmative;

if to *parallel* Rays, $y = \frac{2d\rho}{d} = 2\rho$: So that the Image of a vaſtly diſtant Object is always formed by a *plano-convex Lens*, the plane Side being turned towards the Object, juſt at the Diſtance of the Diameter of the ſecond Surface behind it. A *plano-concave Lens*, expoſed on the *plane* Side to diverging Rays, gives $y = \frac{-2dr\rho}{dr + 2r\rho} = \frac{-2d\rho}{d + 2\rho}$, always negative; to converging Rays, $y = \frac{2d\rho}{-d + 2\rho}$, affirmative, when the *Focus* of diverging Rays on a *plano-convex* is negative, and *vice verſa*; to *parallel Rays*, $y = \frac{-2d\rho}{d} = -2\rho$: So that the Image of a vaſtly diſtant Object is always formed by a *plano-concave* at the Diſtance of the Diameter before the ſecond Surface, as it is by a *plano-convex* behind. A *Meniſcus* expoſed on the *concave* Side to *diverging* Rays, gives $y = \frac{-2dr\rho}{-dr + d\rho + 2r\rho}$, affirmative only when d and r are ſo great in reſpect to ρ, that dr exceeds $d\rho + 2r\rho$: To *converging Rays*, $y = \frac{2dr\rho}{dr - d\rho + 2r\rho}$, affirmative or negative, according as $dr + 2r\varrho$ is greater or leſs than $d\varrho$: If to *parallel* Rays

$$y = \frac{-2dr\varrho}{-dr + d\varrho} = \frac{-2r\varrho}{-r + \varrho},$$

affirmative, if the Radius of *Concavity* is greater than the Radius of *Convexity*; and negative if leſs; and infinite if equal: For the Effects of the firſt Surface are, in that Caſe, exactly deſtroyed by the ſecond, and the Rays ſuffered to proceed ſtill parallel.

It is to be obſerved, that if the Thickneſs of the *Lens* is neglected, as inconſiderable, the *Focus* of all Sorts of Rays falling upon any *Lens* will be exactly the ſame, upon whichſoever Surface of the *Lens* they are firſt received. But if the Thickneſs of the *Lens* be conſidered, there will be ſome Difference in the *focal* Diſtance, according as you turn this or that Surface of the *Lens* towards the Rays. And this Difference is eaſily found from the general Rule: For upon turning the other Surface of the *Lens* towards the Rays, ϱ becomes r, and r changes to ϱ; by which means the Rule will give us the *focal* Diſtance in both Caſes, and ſubſtracting one from the other, we find their Difference. Thus, if, to abridge the Rule, we ſuppoſe the Rays parallel, d being infinite, we have for a *double Convex* in one Caſe $y = \frac{Ipr\varrho - R\varrho t}{Ir - It + I + \varrho Rt}$, and upon turning the *Lens*, $y = \frac{I\rho r - Rrt}{I\rho - It + Ir + Rt}$; wherefore ſubſtracting one from the other, according as ρ or r is greateſt, we ſhall have the Difference in *double Convex*, occaſioned by turning the *Lens* $= \frac{\substack{+Rrt \\ -} \; \substack{-R\rho t \\ +}}{Ir - It + Rt + I\rho}$, or in Glaſs $\frac{\pm 2rt \mp 2\rho t}{3r - t + 3\rho}$. And this is applicable to *Lenſes* of any other Figure, by changing the Signs + or — of thoſe Terms, where we meet with r or ρ, or making them infinite, according as the reſpective Surfaces are *concave* or *plane*. Thus in the Caſe of a *plano-convex*, r being infinite, the Difference ariſing upon turning the *Lens* becomes $\frac{Rrt}{Ir} = \frac{R}{I}t$; or in Glaſs $\frac{2}{3}t$, in Water $\frac{3}{4}t$, and $\frac{2}{5}t$ in Diamond. Which ſhews that the *focal* Diſtance is greater by $\frac{2}{3}t$ when the *plane* Side of a *plano-convex* of Glaſs is turned towards a *vaſtly diſtant* Object, than when the *convex* Side is turned to it. After the ſame manner, the negative *focal* Diſtance of a *plane Concave* will be greater by $\frac{2}{3}t$, when the *plane* Side is turned towards the *vaſtly diſtant* Object, than when the *concave* Side is turned to it. In *double Concaves*,

caves, where the *Focus* is always negative, the Difference is $\frac{\pm 2\rho t \mp 2rt}{-3r - t - 3\rho}$, according as ρ is greater or leſs than *r*. In *Meniſci*, the *focal* Diſtance, whether affirmative or negative; being always greateſt when the *concave* Surface is turned towards a vaſtly diſtant Object, the Difference is $\frac{-2rt - 2\rho t}{-3r - t - 3\rho}$ when the *Foci* upon turning either Side fall both one way, and are either both affirmative or negative: But if *t* be ſo conſiderable as to be greater than 3ρ, the *focal* Diſtance, uupon turning the *concave* Side, $\frac{-6r\rho - 2\rho t}{-3r - t + 3\rho}$ is affirmative, and the *focal* Diſtance, upon turning the *convex* Side, $\frac{-6r\rho + 2rt}{3\rho - t - 3r}$ negative; and conſequently their Difference is $\frac{2rt - 12r\rho - 2\rho t}{-3r - t + 3\rho}$: And if *t* were equal to 3ρ, the *Focus* in this Caſe, upon turning the *convex* Side to a *vaſtly diſtant* Object, falling exactly upon the *Vertex* of the ſecond Surface of the *Lens*, and conſequently the *focal* Diſtance being equal to nothing, the Difference will be the ſame with the *focal* Diſtance upon turning the *concave* Side, namely, $\frac{-6r\rho-6\rho\rho}{-3r}$. After the like manner, may be found from the general Rule, the Difference which would ariſe upon turning the different Surface of any Sort of *Lens* towards other Rays beſides *parallel*, whether *diverging* or *converging*; but the *Canons* for theſe Caſes conſiſt of ſo many Terms, and are of ſo little Uſe, that they are not worth having.

There are three farther *Uſes* to be made of the general Rule above delivered; the firſt is, from the *Lens* or *Focus*, where an Object is repreſented, being given, to determine the *Diſtance* of the Object from the *Lens*; or the *Lens* by which we would form the Image of any Object, and the *Focus* where we would have it formed, being given, to determine the Diſtance at which the Object ſhould be placed before the *Lens*, that it may be repreſented in the given *Focus*: That is, in the Equation $y=\frac{pdr\rho}{dr+d\rho-pr\rho}$, or $dry+d\rho y-pr\rho y=pdr\rho$, r, ρ, p, and y being given, 'tis required to find d, and conſequently we ſhall have $d=\frac{pr\rho y}{ry+\rho y-pr\rho}$; where 'tis plain, that if r and ρ be ſo great in reſpect to y, that $pr\rho$ exceeds $ry + \rho y$, d will be negative; and the Object cannot be repreſented in the Circumſtances required, unleſs by means of another *Lens*, we firſt make the Rays coming from the Object, *diverging* converge to a Point behind the firſt Surface of the *Lens* given, at the Diſtance of

$$\frac{pr\rho y}{pr\rho - ry - \rho y}$$

: And if $pr\rho$ is equal to $ry + \rho y$, d will be infinite. Suppoſe the given *Lens* a *double Convex* of Glaſs, and made of two Segments of equal Spheres, but of a Thickneſs not conſiderable, and it is required to find at what Diſtance from the *Lens* a lucid Body ſhould be placed, in order to have its Beams parallel after their Emerſion from the *Lens*, and conſequently its Light thrown upon Objects *vaſtly diſtant*, which may be thereby illuminated: In this Caſe y being infinite, and r equal to ρ, and p equal to 2, we ſhall have the Diſtance required $d = r$. But if t be conſiderable, we muſt find d from the Rule which takes in the Thickneſs of the *Lens*, which gives us the exact Value of

$$d = \frac{4r\rho t + 64\rho y - 2rty}{3ry - ty + 3\rho y - 6r\rho + 2\rho t}$$

: As if, for Example, the *double Convex* juſt mentioned, were an entire *Sphere* of Glaſs, and the ſame thing required as before, y being, as we have already obſerved, infinite, and *r* equal to ρ, and moreover *t* equal to $2r$; this laſt Rule gives $d = \frac{6rry - 4rry}{3ry - 2ry + 3ry} = \frac{1}{2}r$; whereas by the former, which neglects the Thickneſs, we have the Diſtance required twice as great, or $d = r$; a Difference very conſiderable, if the Spheres be of any Bigneſs. So then, a lucid Body placed at the Diſtance of half the Radius from a Sphere of Glaſs, or at the Diſtance of the whole Radius from a *double Convex* of equal Spheres, whoſe Thickneſs is inconſiderable, will illuminate Objects vaſtly diſtant. If the given *Lens* were a *Hemiſphere* of Glaſs, and the ſame thing ſtill required; if the *convex* Surface be firſt, both *y* and ρ being in this Caſe infinite, and *t* equal to *r*, the Diſtance of the lucid Body will be $d = \frac{64\rho y}{3\rho y} = 2r$; but if the *plane* Side be next the lucid Body, *y* and *r* being infinite, we have $d = \frac{6r\rho y - 2rty}{3ry} = \frac{4}{3}\rho$, or $\frac{4}{3}r$; there being as has been ſhewn before a Difference of $\frac{2}{3}t$ in the *focal* Diſtance of a *plano-convex* expoſed to *parallel* Rays, occaſioned by turning the different Sides of the *Lens*: If *t* had been neglected, we ſhould have had $d = 2r$ in both Caſes. If we have an Object repreſented by a *double Concave* of Glaſs of equal Spheres, at a negative *Focus* the Diſtance of the Radius from the *Lens*, and it were required to find the Diſtance of the Object, y, r, and e being all negative and equal, and t inconſiderable, we ſhall have

$$d = \frac{-2rrrr}{2rr - 2rr} \text{ infinite;}$$

and conſequently the *Object* is *vaſtly diſtant*. The ſame thing may be done for all other Caſes whatever, only remembering to make the proper Alterations according as r, e, or y are negative or infinite, and t conſiderable or inconſiderable.

The ſecond *Uſe* is, from one Surface (either the firſt or ſecond) of a *Lens* being given already formed, to find what Degree of *Convexity* the other Surface muſt have, in order to repreſent a given Object at a given *Focus*: That is, in the Equation before uſed $dry + dey - prey = pdre$, d, y, p and r being given, to find e, or e being given, to find r. Whence we have

$$e = \frac{dry}{pdr - dy + pry}, \text{ and } r = \frac{dey}{pde - dy + pey};$$

which will ſerve for any other *Lenſes* beſides *double-convex*, and any other Rays beſides *diverging* ones, by making ſuch Alterations as have been already directed. If the firſt Surface of a Glaſs *Lens* were *plane*, and it were required to find what Degree of Convexity the ſecond Surface muſt have, in order to repreſent an Object at a *Focus* juſt as far diſtant from the *Lens* as the Object it ſelf; in this Caſe d is equal to y, and r infinite, and conſequently $\rho = \frac{dry}{2pry} = \frac{1}{4}d$: So that the ſecond Surface of the *Lens* muſt be made of a Segment of a Sphere, whoſe Radius is equal to ¼ of the Diſtance of the Object. If the Object to be repreſented at a given *Focus*, be *vaſtly diſtant*, d being in this Caſe infinite, the Rule is abridged to $\rho = \frac{ry}{pr - y}$: Whence 'tis plain, that if in Glaſs y is greater than $2r$, or the given *Focus* be at a greater Diſtance from the *Lens* than twice the Radius of the given Surface, ρ will be negative, and the ſecond Surface muſt be made *concave*; and if y be equal to $2r$, ρ is infinite, and the ſecond Surface muſt be *plane*. If the Thickneſs of the *Lens* be ſo great that it ought to be conſidered, we muſt find ρ from the general Rule.

The third *Uſe* is, from the *Lens*, *Diſtance* of the Object, and *Focus* being given to determine the *Ratio of Refraction*: That is, in the Equation before uſed $dry + d\rho y - pr\rho y = pdr\rho$,

$= pdre$, d, r, e and y being given, to find p, which gives us $p = \frac{dry + dey}{dre + rey}$: For p being found, the Value of $p = \frac{\mathrm{I}}{\mathrm{I} - \mathrm{R}}$ gives the *Ratio* I to R as $\mathrm{I} + p$ to p. If a *double-convex Lens*, made of two Segments of the ſame Sphere, repreſents, or is required to repreſent, a vaſtly diſtant Object, at a *Focus* the Diſtance $\frac{1}{2}r$ from the *Lens*; d being in this Caſe infinite, and r equal to e, and y equal to $\frac{1}{2}r$, we ſhall have $p = \frac{3drr}{drr} = 3$, and conſequently the *Ratio* I to R as 4 to 3; whence the *Lens* is made, or ought to be made, of the *Medium* of Water. If the *focal* Diſtance were $\frac{1}{3}r$, we ſhould have $p = \frac{2}{3}$, or I to R as 5 to 2, and conſequently the *Lens* would be *Diamond*. If the *focal* Diſtance were $4r$, then we have $p = 8$, or I to R as 9 to 8, and the *Lens* is *Glaſs*, and the *ambient Medium Water*. But if we are very curious in determining the *Ratio* of *Refraction*, it is done more exactly when the *Lens* is formed into an *Hemiſphere*, or a *plano-convex*, and receiving the Rays of the Sun upon its *plane* Side, collects them in a Point at ſome Diſtance behind, which muſt be meaſured with great Niceneſs; becauſe in this Caſe our neglecting t occaſions no Error at all.

In this Caſe, if the *focal* Diſtance is equal to thrice the Radius of the Sphere, d and r being infinite, and y equal to $3e$, 'twill be $p+3$, or I to R as 4 to 3; if y is equal to $2e$, I is to R as 3 to 2; and if y is equal to e, then I is to R as 2 to 1.

V. At Prop. XX. Prob. XIII.

WHich is, *To find the Diſtance at which an Object ſhould be placed from a given* Lens, *ſo as that the Image formed by the* Lens *may bear a given Proportion to the Object.*

The Author has given the *Conſtruction*, but omitted the *Demonſtration*; leaving the Proceſs of the Calculation which points out that Conſtruction, as a Trial of Skill to the diligent Reader. But becauſe my Deſign in publiſhing the Book is to make it entirely eaſy; for fear it may prove too difficult or diſcouraging a Task to ſome who are either not skilful enough, or perhaps too lazy, to go through with it, I have ſubjoined the following *Solution*.

[*Plate* III. *Fig.* 6, 7.] Let the given *Lens* BD be a *double Convex*, and call AB the Radius of the firſt Surface a, CD the Radius of the ſecond Surface b, BD the Thickneſs of the *Lens* c, the *Proportion* of the *homologous* Lines of the Object and Image as r to b, and E B the Diſtance required z, at which the Object is to be placed before the *Lens* B, which we ſuppoſe made of Glaſs, and the ambient *Medium* Air. It is plain there are different Values of E B (z) according as the *Focus* F is affirmative, or beyond the *Lens*, as at *Fig.* 6. or negative, and on the ſame Side of the Object, as at *Fig.* 7. Both which Caſes ſhall be reſpectively conſidered, and included in the Demonſtration.

By *Corol. Prop.* XVIII. the *homologous* Lines of the Object and Image are to another as their reſpective Diſtances from the *Lens*; wherefore r is to b as E B (z) the Diſtance of the Object required to D F the *focal* Diſtance of the deſired Image, which is conſequently $\frac{bz}{r}$. But we ſhall have another Value of this *focal* Diſtance from *Prop.* XV. For if we look upon f as the *Focus* of Rays ſent diverging from the Object at E, after their Refraction at the firſt Surface of the *Lens*, and F their *Focus* after both Refractions; and call the firſt *focal* Diſtance Bf, $=x$, and the ſecond D F, $=y$. Before we can find the *focal* Diſtance D F, which determines the Place of the *Image*, we muſt firſt find B f. Now to find B f, by *Prop.*

XV. $\frac{EA}{3}\left(\frac{z+a}{3}\right) : AB\,(a) :: Ef\,(\pm z+x) : Bf\,(x)$; whence $zx+ax=\pm 3az+3ax$, and $Bf\,(x)=\frac{\pm 3az}{z-2a}$. Having found Bf the ſame Propoſition gives us DF, for $\frac{fC}{2}\left(\frac{\frac{\pm 3az}{z-2a}\mp c\pm b}{2}=\frac{+3az\mp cz\pm bz\pm 2ac\mp 2ab}{2z-4a}\right) : CD\,(b) :: fF\left(\frac{3az}{z-2a}-c\pm y=\frac{3az-cz\mp yz\pm 2ac\pm 2ay}{z-2a}\right) : DF\,(y)$; whence $+3azy\pm czy\pm bzy\pm 2acy\mp 2aby=6abz-2bcz\mp 2bzy+4abc\pm 4aby$, and $DF\,(=y)=\frac{6abz-2bcz+4abc}{\pm 3az+cz\pm 3bz\pm 2ac\mp 6ab}$. Therefore comparing the two Values of DF together, we have this Equation $\frac{bz}{r}=\frac{6abz-2bcz+4abc}{\pm 3az+cz\pm 3bz\pm 2ac\mp 6ab}$, or $\pm 3abz^2\mp bcz^2\pm 3bz^2\pm 2abcz\mp 6abz^2=6abrz-2bcrz+4abcr$. From whence we have $zz=\frac{6abz-2acz\pm 6arz\mp 2rcz\pm 4arc}{3a-c+3b}$

And if the Thickneſs of the *Lens* be neglected as inconſiderable, all the Terms where C occurs vaniſhing, we have $zz = \frac{6abz \pm 6arz}{3a + 3b}$, and conſequently E B ($z$) $= \frac{2ab \pm 2ar}{a + b}$. *Q. E. I.*

[*Plate* II. *Fig.* 22.] If then we would *conſtruct* this Equation, we have this Proportion given us for that Purpoſe, $a + b : b \pm r :: 2a : z$. Taking therefore A B equal to a, and B C in the ſame right Line equal to b; from C draw at pleaſure the indefinite right Line C M, upon which cut off C D equal to C B (b), and from D on either Side take D M, or D m ſuch, that it may bear the ſame *Proportion* to D C, which the *homologous* Lines of the Object do to thoſe of the Image. Join A M, or A m, to which thro' B draw B E, or B e, parallel; and twice M E, or twice me, is the *Diſtance required* in the *Problem*. For A C ($a+b$): M C, or m C ($b \pm r$) :: 2 A B ($2a$) 2 M E or 2 me (z). And conſequently the *Conſtruction* gives the true Value of z, as before found. *Q. E. D.*

The ſame *Problem, of magnifying or diminiſhing a given Object by a given* Lens, *in any aſſigned Proportion*, may be ſolved from the Equation above given, for finding the *Foci* of all ſorts of *Lenſes*, which if expreſſed in the Characteriſticks in preſent Uſe, is $y = \frac{pabz}{az+bz-pab}$. For ſuppoſing as before, r to b expreſſes the Proportion which the *homologous* Lines of the Object are required to bear to thoſe of the Image, and 'tis deſired to find z the Diſtance of the Object from the *Lens*, which is neceſſary to perform the Conditions required: By *Corol. Prop.* XVIII. we have another Value of the *focal* Diſtance $y = \frac{bz}{r}$. Whence comparing both together we have $\frac{bz}{r} = \frac{pabz}{az+bz-pab}$, and conſequently if the *Focus* is to be affirmative $z = \frac{pab+par}{a+b}$; but if pab be greater than $az+bz$, then the *Focus* is negative, or on the ſame Side with the Object, and this negative *focal* Diſtance is $\frac{pabz}{-az-bz+pab}$, and conſequently $z = \frac{pab-par}{a+b}$: That is, if the *Lens* be Glaſs, the following Equation includes both Caſes, whether the Image is to be repreſented on the contrary or ſame Side with the Object, $z = \frac{2ab \pm 2ar}{a+b}$. Where it is to be obſerved, that if the Image is repreſented on the contrary Side by a *double-convex Lens*, or at an affirmative *Focus*, it may be made either equal to, greater or leſs than the Object in what Proportion we pleaſe; but if it is repreſented on the ſame Side, or at a negative *Focus*, r muſt always be leſs than b, and conſequently the Image may be ſhewn larger than the Object in all the Degrees imaginable, but never leſs, nor equal; for when b is equal to r, and the *Focus* negative, z is $=o$, and when b is leſs than r, z is negative and impoſſible.

Nor is this Solution confined to the Caſe of *double Convexes* only, tho' made for *Lenses* of that Figure, but will with proper Alterations extend equally to *Lenses* of all other ſorts whatever; only obſerving to change the Signs + or — with which the Radius of a *concave* Surface is affected, or making the Terms infinite where the Radius of a *plane* Surface occurs; becauſe it has been ſhewn before, that the Radius of a *concave* Surface bears a contrary Sign to that of a *convex*, and the Radius of a *plane* Surface is infinite. If then the *Lens* be a *concavo-convex*, or *Meniſcus* of Glaſs, and the firſt Surface concave, the Rule for both Caſes, whether the *Focus* is to be affirmative or negative, is $z=\dfrac{-2ab\pm 2ar}{-a+b}$; and if the ſecond Surface be that which is *concave*, then for the affirmative and negative *Foci* reſpectively, the Rule becomes $z = \frac{-2ab \pm 2ar}{a-b}$. Where it is to be obſerved, that in the firſt Caſe if the *Focus* be affirmative, *a* muſt be greater than *b*, or elſe *z* will be negative, and the Problem impoſſible; and if the *Focus* be negative, and *b* greater than *r*, then muſt *a* be ſtill greater than *b*; but if *b* be leſs than *r*, then *a* muſt be leſs than *b*, or elſe the Problem will be impoſſible: And in the laſt Caſe, if the *Focus* be affirmative, and *b* greater than *r*, then *b* muſt be greater than *a*, and *vice verſa*; and if the *Focus* be negative, *b* muſt ſtill be greater than *a*. Which ſhews that in the Caſe of a *Meniſcus* turned on the *concave* Side towards the Object, the Image can never be repreſented at all on the oppoſite Side, unleſs the *concave* Surface be a Segment of a larger Sphere than the *convex*, and then it may be ſhewn in what Proportion to the Object we pleaſe; and if it is to be repreſented on the ſame Side with the Object, and magnified, the Radius of *Concavity* muſt be ſtill larger than that of *Convexity*, and *vice verſa*, if the Image is to appear diminiſhed. And in like manner may be underſtood what will happen upon turning the convex Side of the *Meniſcus* towards the Object. If the *Lens* be a *double concave* of Glaſs, the *Focus* being in this Caſe always negative, we have but one Value of z, which is $\frac{2ab-2ar}{-a-b}$, affirmative only when r is greater than b; which ſhews that a *double-concave* can only diminiſh. If the *Lens* be a *plano-convex* of Glaſs, it will be $z=2b\pm 2r$; which ſhews that in this Caſe the Object may either be magnified or diminiſhed, if the *Focus* be affirmative, but only magnified if it be negative. It muſt be noted, that if the ſecond Surface, whoſe Radius is b, were ſuppoſed plane, we ſhould have $z=2a$; becauſe b not only ſtands for the Radius of that Surface, but alſo expreſſes the *Ratio* of the Image to the Object, which is conſequently in this Caſe infinite, and the Image *vaſtly diſtant*. If the *Lens* be a *plano-concave* of Glaſs, the *Focus* being always negative, we have only one Value of z, which is $z=-2b+2r$; which ſhews, that a *plano-concave* can only diminiſh.

It may be remarked, that thoſe Caſes which make the Diſtance z negative, and the Problem impoſſible for the given *Lens*, may by means of another *Lens* be made practicable: If we firſt receive the Rays of the Object upon this ſecond *Lens*, and before

fore they are collected at the *Focus* throw them upon the given *Lens*, in ſuch a manner as to make them fall *converging* to a Point behind the firſt Surface of the given *Lens*, at the Diſtance of the negative Value of z.

If the *Lens* were a *double-convex* of *Water*, p being $=\frac{R}{1-R}$, it is $z=\frac{3ab\pm 3ar}{a+b}$; if of *Diamond* $z=\frac{\frac{2}{3}ab\pm\frac{2}{3}ar}{a+b}$; if of *Glaſs* in an ambient *Medium* of *Water* $z=\frac{8ab\pm 8ar}{a+b}$; if of *Diamond* in an ambient *Medium* of *Water* $z=\frac{\frac{2}{7}ab\pm\frac{8}{7}ar}{a+b}$, and conformably in all other Caſes.

If the Image be deſired juſt as great as the Object, then for a *double-convex* of Glaſs, r being equal to b, the Rule gives the Diſtance required $z=\frac{4ab}{a+b}$; where it is plain the Problem is only poſſible when the *Focus* is affirmative. If the *Lens* were equally *convex*, b being in this Caſe equal to a, there will only be one Value of z, and that for an affirmative *Focus* $z=2a$: So that if the Object be placed at twice the Diſtance of the Radius from the *Lens*, the Image formed at its *Focus* will be juſt as great as the Object. If the *homologous* Lines of the Image were deſired twice as large as thoſe of the Object, r being in this Caſe equal to $\frac{1}{2}b$, for a *double-convex* of Glaſs the Rule gives

$$z=\frac{2ab\pm ab}{a+b},$$

and if the *Lens* be equally *convex* $z=3a$ or $\frac{1}{2}a$, according as the *Focus* is affirmative or negative, ſo that an Object placed at the Diſtance either of $\frac{3}{2}$ or only $\frac{1}{2}$ the Radius from the *Lens*, is repreſented at a *Focus*, either affirmative or negative, twice as large every way as the Object, or the Image in its whole Content will be four times as large as the Object. If the *homologous* Lines of the Image were deſired twice as ſmall as thoſe of the Object, r being in this Caſe equal to $2b$, the Rule gives for an equally convex *Lens* only one Value of the Diſtance $z=3a$, a *double-convex* being only capable of repreſenting an Object diminiſhed, when the *Focus* is affirmative. If the Image be deſired an hundred times larger than the Object, or its *homologous* Lines ten times as large, r being in this Caſe equal $\frac{1}{10}b$, if the *Lens* be equally *convex*, the Rule gives the Diſtance $z=1\frac{1}{10}a$, or $\frac{9}{10}a$, according as the *Focus* is affirmative or negative. And in general if m to n expreſs the Proportion which r bears to b, the Rule laid down at firſt $\frac{pab\pm par}{a+b}$, becomes

$\frac{pab \pm p \frac{m}{n} ab}{a + b}$, and if the *Convexity* be equal, it is $z = \frac{1}{2} pa \pm \frac{1}{2} p \frac{m}{n} a$. From all which it appears, that with a *Lens* equally convex on both Sides, in order to magnify a given *Object*, the Diſtance is always ſomething greater than $\frac{1}{2} pa$, when the *Focus* is affirmative, and always leſs than $\frac{1}{2} pa$ when the *Focus* is negative, unleſs the *Ratio* of the Image to the Object be infinitely great, and then n being infinite, 'tis $z = \frac{1}{2} pa$: That is, in Glaſs the Diſtance of the Object muſt always exceed the Radius, if the *Focus* is to be affirmative, or fall ſhort of it if negative, and be equal to it when the Image is to be infinitely great, or vaſtly diſtant. And in order to diminiſh a given Object $\frac{m}{n}$ in this Caſe exceeding *Unity*, the Diſtance, which has but one Caſe here, is always greater than pa; and the more z exceeds pa, the more the Object is diminiſh'd, and *vice verſa*, 'till z becoming equal to pa, the Object and Image are likewiſe equal.

All this is eaſily obſerved in that common Experiment of the *Camera obſcura*; where the Rays propagated from external Objects are received by a *Lens*, and tranſmitted into the Room, and do there paint, upon a white Sheet placed at the *focal* Diſtance of thoſe Rays from the *Lens*, the Images of their reſpective Objects, in Colours ſcarce leſs lively than thoſe of the Objects themſelves. And hence this Problem of magnifying or diminiſhing a given Object, may be of great Uſe in *Painting*; for by admitting the Image of any Object by means of a *Lens* into a dark Chamber, in what *Ratio* to the Life we pleaſe, 'tis eaſy to hit the Proportion of every Part with great Exactneſs, which is otherwiſe but ſeldom done, eſpecially in ſuch Pieces where the Figures are either much greater or much leſs than the Life. 'Tis true, theſe Images that are repreſented in the *dark Chamber* by a ſingle *Lens* appear inverted, but may be made erect by uſing a ſecond *Lens* after the following manner: Place the *Object* at ſuch a Diſtance from the firſt *Lens* in the Window, that the *Image* formed by that may be juſt as big as the Life; then beyond the Place of this *Image* fix the given *Lens* at the Diſtance required *z*; this will form an *Image* of the former *Image* which ſhall be in the *Ratio* aſſigned, and alſo erect, but ſomething leſs lively. What has been above delivered is likewiſe of Service in the Conſtruction of the *Magick Lantern*, and other optical Machines, where the *Images* of any *Objects* are to be repreſented monſtrouſly larger or leſs than the Life.

VI. At Prop. XXIV. Prob. XVII.

WHich is, *To make a concave* Speculum *of Glaſs of a given Thickneſs*; *the Radius of whoſe Concavity is alſo given, in ſuch a manner, that parallel Rays reflected from the firſt Surface of the* Speculum *may meet in the ſame Point of its* Axis *with thoſe that paſs refracted into the* Speculum, *and are refracted from the ſecond Surface, and again refracted at their Emerſion from the* Speculum. Or the firſt Surface of a *concave Speculum* being already formed, to determine of what Sphere 'tis neceſſary to take a Segment to form the ſecond Surface in ſuch manner, that an *Object vaſtly diſtant* may be repreſented by Reflexion from both Surfaces in one and the ſame Place, or that the two Images may be united, and conſequently be made more lively. The Author has given two ſeveral *Conſtructions* of the *Problem*; the firſt exact, and the other only near the Truth, but more expeditious: But for Reaſons already mentioned he has omitted their *Demonſtration*, which is as follows.

[*Plate* III. *Fig.* 8.] Let A B be the given Radius of the firſt Surface, and B D the given Thickneſs of the *Speculum*, and C D the Radius required of the hindmoſt Surface ſace neceſſary to perform the Conditions of the *Problem.* Suppoſe the Point f in fD, the Axis of the *Speculum* to be the *Focus* of the *parallel Rays*, after *Refraction* at their Entrance into the firſt Surface, φ their *Focus* after *Reflexion* from the ſecond Surface, and F their *Focus* after *Refraction* again at their Emerſion from the *Speculum*; 'tis required that the Point F ſhould be the ſame with the *Focus* of the ſame *parallel Rays* after their *Reflexion* from the firſt Surface. Call the given Radius A B a, the given Thickneſs B D c, the firſt *focal* Diſtance Bf v, the ſecond $D\varphi$ x, the laſt *focal* Diſtance B F y, and the required Radius of the hindmoſt Surface C D z. We muſt find theſe *focal* Diſtances one after another, in order to determine the laſt B F, which muſt be equal to the *focal* Diſtance of *parallel Rays* reflected from a *concave Speculum.* For the firſt then Bf, by *Prop.* XIV. $Bf\ (v) : fA\ (v-a) :: I : R ::$ (in Glaſs) $3 : 2$; whence $Bf\ (v) = 3a$. And for the ſecond $D\varphi$, by *Prop.* IV. $D\varphi\ (x) : \varphi C\ (z-x) :: Df\ (3a+c) : fC\ (3a+c-z)$; whence $3ax+cx-zx=3az+cz-3ax-cx$, and conſequently

$$D\varphi\ (x) = \frac{3az+cz}{6a+2c-z}.$$

Laſtly, to determine the *focal* Diſtance B F after both *Refractions* at the firſt Surface, and the *Reflexion* from the laſt, by *Propoſit.* XV. the Emerſion being out of

Glaſs into Air, $\frac{\varphi A}{2}\left(\frac{a+c-\frac{3az+cz}{6a+2c-z}}{2}\right.$ $= \left.\frac{6aa+8ac-4az+2cc-2cz}{12a+4c-2z}\right)$: AB (a) : : $\varphi F\left(\frac{3az+cz}{6a+2c-2}-y-c=\right.$ $\left.\frac{3az+2cz-6ay-2cy+zy-6ac-2cc}{6a+2c-z}\right)$: BF (y); whence $6aay+8acy-4azy+2ccy-2czy=6aaz+4acz-12aay-4acy+2azy-12aac-4acc$, and conſequently BF $(y)=\frac{3aaz+2acz-6aac-2acc}{9aa+6ac-3az+cc-cz}$.

Now this muſt be equal to the *focal* Diſtance of *parallel Rays* reflected from a *concave Speculum*, which by *Prop.* III. is juſt half the Radius; whence we have another Value of BF $(y)=\frac{a}{2}$. And comparing both together, we have $\frac{3aaz+2acz-6aac-2acc}{9aa+6ac-3az+cc-cz}=\frac{a}{2}$, or $9az+5cz-18ac-5cc=9aa$, and conſequently the Radius required CD $(z)=\frac{9aa+18ac+5cc}{9a+5c}$. *Q. E. I.*

Fig. 3. If then we would *conſtruct* this Value of CD (z), we have the following

Proportion given us for that Purpoſe $9a+5c : \sqrt{9aa+18ac+5cc} :: \sqrt{9aa+18ac+5cc} : z$. Wherefore making the rectangular Triangle MLK in ſuch manner, that LM ſhall be equal to $9a+5c$, and LK equal to $\sqrt{9aa+18ac+5cc}$; and then drawing from K the Perpendicular SK, the right Line LS is the Radius required. For by *Element*. VI. 8. LM $(9a+5c)$: LK $(\sqrt{9aa+18ac+5cc})$ :: LK $(\sqrt{9aa+18ac+5cc})$: LS (z). And conſequently the *Conſtruction* gives the true Value of CD (z) as before found. *Q. E. D.*

Fig. 4. The Radius required is alſo capable of another *Conſtruction*; for making an actual *Diviſion* of $9aa+18ac+5cc$ by $9a+5c$, the *Quotient* is $a+c+\frac{4ac}{9a+5c}$: And if AB (a) be ſufficiently great in reſpect to BD (c), the Term $5c$ in the *Denominator* of the *Fraction* may be neglected; and then it becomes $z=a+c+\frac{4}{9}c$, whoſe Exceſs above the Truth is not at all ſenſible. Wherefore taking DO equal to $\frac{4}{9}$ of BD (c), and making AC equal to DO, CD $(a+c+\frac{4}{9}c)$ is very nearly the true Value of the *Radius required*. *Q. E. D.*

To ſhew how near this laſt Value of $z=a+c+\frac{4}{9}c$ is to the Truth, if we ſuppoſe the Thickneſs of the *Speculum* to be $\frac{1}{5}$ of the given Radius of the firſt Surface, which is very conſiderable: In this Caſe the exact value of $z = a + c + \frac{4ac}{9a + 5c}$ is but $\frac{2}{225}$ a or nearly $\frac{1}{112}$ a leſs than its Value found by neglecting $5c$ in the other Equation $z = a + c + \frac{4}{9}c$, a Difference not at all conſiderable in *phyſical* Matters.

If c be ſuppoſed equal to $\frac{1}{3}a$, which is ſtill a far greater Suppoſition, even in this Caſe, the Value of the Radius z taken from the laſt Equation $z = a + c + \frac{4}{9}c$, is but $\frac{11}{268}$ a or very nearly $\frac{1}{42}$ a greater than the Truth; which is an Exceſs not very ſenſible, unleſs a be extremely great. But if the Thickneſs be greater than in this laſt Suppoſition, it will be convenient to take the Value of the Radius required from the exact Equation $z = a + c + \frac{4ac}{9a + 5c}$: As, if c be equal to $\frac{1}{2}a$, the Radius required is $1\frac{31}{46}a$; which is about $\frac{1}{21}a$ leſs than what it would be if $5c$ were neglected. If c be equal to a, the Radius required is $2\frac{2}{7}a$; which is about $\frac{1}{6}a$ leſs than it would be if $5c$ were neglected. If the Thickneſs be ſo conſiderable as to be equal to thrice the given Radius of the firſt Surface, then the Radius required is $4\frac{1}{2}a$.

Hence likewiſe, if the Radius of the laſt Surface be given, together with the Thickneſs of the *Speculum*, we may find the Radius of *Concavity* neceſſary to unite the two Images of a vaſtly diſtant Object made by *Reflexion* from both Surfaces. For if c be not very great, we ſhall have $a = z - c - \frac{4}{9}c$, as near the Truth as need be required in Practice: For if we had the exact Value of a, we could not in Practice grind the *Speculum* to the due Concavity, even ſo near as the Value juſt now given. If c be conſiderable, the Value of a muſt be found from an Equation of an higher Degree $9aa - 9az + 18ac = 5cz - 5cc$; which if it be contracted, by putting p for $2c - z$, will give $a = \mp \frac{1}{2}p \pm \sqrt{\frac{5cz - 5cc}{9} + \frac{1}{4}pp}$, the Sign of $\frac{1}{2}p$ being either — or +, according as $2c$ is greater or leſs than z. After the ſame manner, having the Radius of both Surfaces given, we may find what Thickneſs of the *Speculum* is neceſſary to unite the two Images of a *vaſtly diſtant Object*, formed by *Reflexion* from both Surfaces, by means of the following Equation, $5cc + 18ac - 5cz = 9az - 9aa$; which if it be contracted, by putting q for $3\frac{3}{5}a - z$, will give $c = \mp \frac{1}{2}q \pm \sqrt{\frac{9az - 9aa}{5} + \frac{1}{4}qq}$.

VII. A more particular Account of MICROSCOPES and TELESCOPES, from Mr. *Huygens*.

PROP. I.

TO explain the Effects and Uſes of ſingle Microſcopes, *and the Manner of making little* Spheres *and* Lenſes.

[*Fig.* 9, 10.] Let N be the *Lens* QRQ, the Object at its *Focus* R, O the Eye very near the *Lens*. The Rays coming from R will after their Refraction fall parallel upon the Eye, and conſequently make diſtinct Viſion. For the Fabrick of the Eye, having its *focal* Diſtance juſt at the Bottom of it upon the *Retina*, requires that the Rays from each ſingle Point ſhould fall nearly parallel in order to be there collected; that is, that the Baſis of each Cone of Rays flowing from every Point of any Object, which Baſis is the Pupil of the Eye, ſhould bear ſo ſmall a Proportion to the Length of the Cone, as that thoſe Cones may be looked upon as little Cylinders. (The Diſtance indeed requiſite for diſtinct Viſion is not limited to a Point, but is indulged in larger Bounds; becauſe Nature has furniſhed us with the Power of contracting the Pupil as the Object comes nearer, and ſo diminiſhing the Baſis of each Cone in proportion, and conſequently of preſerving diſtinct Viſion; but this is only to a certain, and that no very great Degree.) But the Object QRQ will appear in the ſame Magnitude as if the *Lens* N were removed, and a Plate with a ſmall Hole in it ſubſtituted in its Place, namely under the Angle QAQ. So that all that the interpoſed *Lens* does in this Caſe, is only making diſtinct Viſion, which would without the *Lens* be confuſed. But ſince at the Diſtance ſuppoſe of 8 Inches from the Object, a naked Eye has then diſtinct Viſion; the apparent Image may be ſaid to be ſo much magnified as thoſe 8 Inches exceed the little Space N R, or the *focal* Diſtance of the *Lens* N: Which if it be equal to $\frac{1}{5}$ of an Inch, the Appearance of the Image ſeen diſtinctly by the *Microſcope* is to that ſeen diſtinctly by the naked Eye, as 40 to 1. Therefore the leſs the *focal* Diſtance of the little *Lens* N is, the greater will its Effects be in dilating the Image of a ſmall Object; though there are ſome Inconveniences (to be mentioned afterwards) which here offer themſelves, and forbid our going beyond ſome certain Limits. And the ſame thing happens to little *Spheres*, which may be uſed for *Lenſes*, and might otherwiſe be made as little as we pleaſe. But theſe ſmall *Spheres* are inferior to little *Lenſes* upon this Account, that for the ſame Degree of magnifying, if both be made of Glaſs, the *Lenſes* are three times more diſtant from the Object than the *Spheres*; and by that means leave a ſufficient Space for the lateral Light to enter, and make the Colours of the Object viſible; whereas otherwiſe we are forced to turn the *Microſcope* directly againſt the Light, and can only then diſcern diſtinctly ſuch Objects as by their Thinneſs are pellucid.

The Effects of a little *Sphere*, and what has been ſaid concerning the three times leſs Diſtance, is thus demonſtrated. *Fig.* 11. 9.

Let there be a Glaſs *Sphere* whoſe Center is K, and its Axis A B, in which produced on both Sides the Eye is placed at D, and the Object at C, each of the Diſtances A D, B C being taken equal to half the Radius A K: And conſequently the Point C is the *Focus* where Rays falling parallel to the Axis A B, upon the *Sphere* at A H are after Emerſion collected; as is ſhewn in Article IV. of this *Supplement*. Wherefore an Object placed at C will ſend Rays upon the *Sphere*, which will after Refraction be received parallel by the Eye, and conſequently make diſtinct Viſion. But by *Prop.* XIV. if we take the Point F ſuch, that F A may be equal to the Radius A K, the Point F is the *Focus* towards which parallel Rays after Refraction at the firſt Surface B G do tend in their Paſſage through the *Sphere*, and from which they are diverted after Refraction at their Emerſion, and collected at D. Make G E parallel to the Axis, and comprehending the Portion of the Object C E, and draw the right Line E D. The Ray then E G being refracted at G, proceeds according to G F; and being again refracted at H, goes on to meet the Eye at D. Wherefore the Line C E is ſeen under the Angle A D H, which would appear to the naked Eye under the Angle C D E; which I ſay is but the half of the former.

For becauſe A F is double of A D, the Angle A D H is double of A F H. But D F is parallel to E G, becauſe G E is both parallel to F D, and to be looked upon as equal to it, or to the right Line B C; becauſe C E is to be a Line very ſmall with reſpect to the Diameter of the *Sphere*. Therefore the Angle A D H is alſo double of the Angle C D E, and conſequently equal to the Angle C K E. From whence it is plain, that to the Eye placed at D the Line C E will appear under the ſame Angle in which it would appear to the naked Eye ſeeing from the Point K. Whence if the Diameter of the little *Sphere* A B were $\frac{1}{12}$ of an Inch, we ſhould have K C $= \frac{1}{16}$ of an Inch; which is to the Diſtance of 8 Inches in the Proportion of 1 to 128: So that the increaſed Magnitude of the Object would be as 128 to 1; which is indeed very conſiderable. But if N R, the *focal* Diſtance of the *Lens*, be equal to the right Line K C, we have ſhewn that by this means the Object R Q would be ſeen in the ſame Magnitude as if the Eye were placed at N without the *Lens*; nor in the uſing this *Lens* will the apparent Magnitude be any ways changed, in whatſoever Part of the Axis R N produced the Eye be placed. Therefore 'tis plain the ſame Degree of Magnifying, and the ſame Effect every way, is performed equally by the *Lens* N and the little *Sphere* A B. And it is moreover manifeſt, that the Diſtance R N, being taken equal to K C, is equal to thrice B C. *Q. E. D.*

We are next to explain the Manner how little Spheres *and* Lenſes *may be prepared and fitted for Uſe.*

The leſs *Spheres* are to be, the eaſier they are prepared, after the following manner. Take the ſmalleſt Fragments of Glaſs, and hold them to the lower part of the Flame of a Candle, where the bluiſh Colour is diſcernible, that they may grow red hot; and then if they be taken up by the fineſt Steel-Wire that can be got, and dexterouſly turned, they will be formed into Globules, which are large enough if equal to a Grain of Muſtard-Seed. Out of ſeveral thus prepared, you will find ſome very good; which may be tried by including them in a Braſs-Plate, and is thus done:

Take a Plate of the thinneſt Braſs the Breadth of a Finger, and twice as much in Length, and bend it double; perforate this Rectangle in the middle with the Point of a Needle, and rub the oppoſite Holes ſmooth with a Whetſtone, that no Roughneſs may remain about the Edges, and black them with the Smoak of a Candle, that no Brightneſs may continue within. Put the little *Sphere*, ſtill adhering to the Steel-Wire, into the Holes within the Braſs-Plate, and fix it there, by faſtening the two Leaves of the Plate together. After this manner you may make ſeveral *Microſcopes* with great Eaſe, out of which you may make choice of thoſe that are beſt.

The principal Uſe of this ſort of *Microſcopes* is, to look at Corpuſcles that are pellucid. And they are placed in a *Machine* made in ſuch manner, that by turning a Screw they may approach to or recede from the Object, and ſo be brought to the due Diſtance, which is requiſite for diſtinct Viſion. And to this it conduces very much, that the too great Light be reſtrained, and only admitted through the Hole, which is about four times the Diſtance from the Object. For by this means the Aperture of the *Lens* is better limited than by the Breadth of the contiguous Hole, which there is no Neceſſity at all of ſtraitening. The Eye muſt be brought as cloſe to the little *Sphere* as may be, that it may comprehend the greater Space.

The Corpuſcles or Drops of Liquors which are to be looked at, are put upon a little circular Plane of Glaſs, which is made to ſlide laterally every way, that we may bring every Part of the Object to be viewed ſucceſſively. Some attract the Liquor to be examined into *capillary* Tubes of Glaſs, ſo ſmall as ſcarce to admit an Hair, which has likewiſe its Uſes. But in uſing thoſe little *Lenſes* before mentioned, Care muſt be taken, that while by means of another *Lens* on one Side we caſt Light upon the Object, the Hole of the Aperture may be exactly limited, by trying how much it may lie open without being an Hinderance to diſtinct Viſion. For here the Points of Corpuſcles emit Rays of Light, and are ſo many *radiant* Points, which is quite otherwiſe in thoſe pellucid Corpuſcles that are looked at through little *Spheres*, where the Objects intercept the Light, not emit it.

The Effects of this ſort of little *Lenſes* and *Spheres* are very wonderful, as may be ſeen from thoſe Experiments with them which have been made publick, and from which our Knowledge of Nature has received very great Light and Information. By theſe the circulatory Motion of the Blood has been put beyond Controverſy, which our *Lewenhoek*, the moſt dilligent Obſerver of theſe Matters, has ſhewn me in the Tail of an Eel, to my very great Satisfaction and Delight. For the Blood appears pellucid, and conſiſting of reddiſh Globules, and runs through the Channels of the Arteries, which are continued to the Veins with a very rapid Motion. Which without doubt might be obſerved in all other Animals, if we could find out ſuch Parts in them as are pervious to the Light. He put the live Eel into a Glaſs Tube half full of Water, to which he externally applied the *Microſcope* at that Part where the Extremity of the Tail touched the Tube.

'Tis alſo very pleaſant to obſerve the *Animalcula* that ſwim in Drops of Water, in which we have infuſed Ginger, Pepper, or ſomething elſe of an hot powerful Odour for ſome Days: They are of various Forms, and ſome leſs than others; their Motions are wonderful, and for their Size ſufficiently quick; nor is the Inſtrument apparent by which they perform them, for they have neither Legs nor Arms, nor do they bend their Bodies like Fiſhes. For the little Eels in Vinegar, which are much larger than they, ſwim like thoſe in the River, in which it is very much to be wondered that they ſhould generate little ones of themſelves. For I ſaw one which had four young ones within it (for they are altogether pellucid) and after it had been kept in the Tube for ſome Hours, brought them all forth, every one of which did afterwards ſwim by it ſelf.

It is very probable that thoſe *Animalcula* which I have ſaid move about in Water, are invited thither out of the Air by the Odour of the Infuſion. For the ſame Figures appear upon macerating ſeveral things in Water; but if the Veſſel be covered, none at all appear. Nor is it difficult to conceive how they ſhould be ſupported in the Air, when they are ſo much ſmaller than the fineſt Duſt that is. So that perhaps we draw many thouſands of them into our Lungs every time we fetch our Breath, without knowing it. Nor would it be uſeleſs to obſerve at what time of the Year they appear in greateſt Numbers, and whether they encreaſe in a vitiated Air. Milk appears to conſiſt of ſmall pellucid Globules ſwimming in a Liquor likewiſe pellucid, but of a different Refraction; and hence it is that it appears white, though it contains no other Matter but what is perfectly tranſparent, and without Colour.

I omit thoſe many wonderful Forms of minute Inſects; the Wings of Butterflies and Gnats, covered with little Feathers; the Powders obſerved in the Middle of Flower-Tops, which are nothing elſe but little tranſparent Bladders, filled with that Matter of which the Bees make their Hony, and which they carry between their Legs into their Hives. But what ought to be looked upon as the moſt wonderful and aſtoniſhing of all is, that an immenſe Multitude of *Animalcula* are diſcovered to ſwim in the *Semen maſculinum*, after the manner of little Fiſhes, almoſt of the ſame Figure with a Frog newly formed, and yet without Legs. Which *Animalcula*, I make no queſtion, enter the *Ova muliebria*, and are the Rudiments of what is brought forth from thence. There are ſeveral Conſiderations which confirm this Opinion; nor is it any great Objection, that out of ſo great a Multitude either few or only one of them comes to Maturity, and grows to be an Animal; ſince the ſame abundance and ſuperfluous Fruitfulneſs is equally obſervable in moſt Seeds of Trees and Herbs, as of Firr, Poppies, *&c.*

Theſe *Animalcula*, by reaſon of their wonderful Smallneſs (for even ten thouſand of them are not equal to the ſmalleſt Grain of Sand) ought to be looked at through ſuch Glaſs Globules as have the greateſt Power of Magnifying.

Prop. II.

To explain the Effects of Compound MICROSCOPES.

WE come now to ſpeak of *compound Microſcopes*, by the Help of which ſuch Objects as are not tranſparent are looked at, and their true Colours diſcovered, and that much better and more commodiouſly than through ſingle *Lenſes*.

[*Fig.* 12. 13.] Suppoſe the *Microſcope* be a double one, conſiſting of two *Lenſes*, one leſs A, and another greater B. Why we diſpoſe them ſo, we ſhall afterwards explain. And let B be the ocular *Lens* neareſt the Eye, placed ſuppoſe at C; A the Object *Lens* neareſt the Object placed ſuppoſe at E; and A B C the common Axis of both *Lenſes*. There will be two Caſes, as may be ſeen repreſented in the two Figures to which this *Prop.* refers. In the firſt, Rays proceeding from a ſingle Point E of the Object, and falling upon the *Lens* A, are refracted by it and again united in the Point P, and there interſecting one another, and proceeding towards the *Lens* B, are by it refracted and made parallel, and ſo enter the Eye at C, and by that means make diſtinct Viſion. 'Tis neceſſary therefore that A E, the Diſtance of the Object, ſhould be greater than A Q, the *focal* Diſtance of the *Lens* A. And the *Focus* P muſt be found by *Prop.* XVI. or by making E Q, E A, E P, in a continual Proportion. But the *Lens* B is to be ſo placed, that its *Focus* on that Side towards A may fall exactly upon the Point P, in order that the Rays may be made parallel after Refraction at the *Lens* B. All which is eaſily done by what has been before demonſtrated. The other Figure (13) repreſents the ſeveral Rays DAG, FAH, EAB, proceeding from different Points of the Object. A is the middle Point of the *Lens*, and A P, A B, AC, are made in a continual Proportion in order to determine the Place of the Eye C; for by this means, however ſmall the Aperture of the little *Lens* A may be, the whole *Lens* B will nevertheleſs be filled with the Image of the Object, becauſe the Rays falling from A upon the whole *Lens* B are collected in the Point C.

[*Fig.* 13.] But the Proportion of the apparent Magnitude to the true will be found by drawing the right Line CF. For the Proportion required will be the ſame to that which the Angle BCH bears to the Angle ECF; which Proportion is compounded of the Proportion of the Angle BCH to BAH, and of the Angle BAH or EAF to the Angle ECF. But the firſt of theſe is the ſame with the Proportion of the right Line AB to BC, and the laſt that of CE to

EA;

E A; becauſe Angles that are ſmall are looked upon to be to one another as their Tangents. Therefore the Proportion of the apparent Magnitude to the true will be compounded of the Proportions A B to B C, or A P to P B (for A P, A B, A C, are in a continual Proportion) and C E to E A. But that the Effects of the *Microſcope* may be more exactly eſtimated, the Angle BCH is rather to be compared with the Angle under which the right Line E F would be ſeen at the Diſtance of 8 Inches from the Eye; that is, with the Angle E LF, LE being taken equal to 8 Inches, according to what has been ſaid before of magnifying by a ſingle *Lens*. And conſequently the Proportion of magnifying muſt be here underſtood to be compounded of the Proportion of the Angle BCH to BAH, and BAH or EAF to E LF; that is, of the Proportion of A P to P B, and of E L, a Line 8 Inches long, to the right Line E A. For if the *Microſcope* were of ſo great Length, that, for Example, C E ſhould be two Foot long, that is, equal to thrice the right Line L E, and the apparent Magnitude to the true were found by the former Reaſoning to be as 90 to 1; yet it is not really any greater than as 30 to 1, becauſe the right Line E F would only appear 30 times greater by the Aſſiſtance of the *Microſcope*, than it would if viewed by the naked Eye at the Diſtance of 8 Inches:

For we are not to conſider how much, by means of the *Microſcope*, we magnify an Object at the Diſtance of two Foot; but how much greater it is made than when viewed at that Diſtance to which we bring our Eye when we deſire to look at any thing more curiouſly.

Of the LIGHT *and* APERTURE *of* MICROSCOPES.

UPon the *Aperture* of *Microſcopes* all their Effects and Virtue entirely depend: So that from hence it is that we are to learn to what Degree the magnifying of Objects may be brought; which no body that I know of has hitherto determined. And it will be found that we may here proceed *ad infinitum*, as ſhall be ſhewn in *Teleſcopes*, not indeed in a ſingle *Microſcope* of one little *Lens*, but in thoſe which are made by a Combination of more than one.

In *Microſcopes* made of a ſingle *Lens* it is to be obſerved, that if their *focal* Diſtance be about half an Inch or greater, there will be no occaſion for limiting the *Aperture* in order to make diſtinct Viſion; becauſe the very Narrowneſs of the Pupil of the Eye excludes, as much as there is occaſion, thoſe Rays that diſturb Viſion, and as much as they would be excluded if the *Lens* were made to have a leſs *Aperture.* But in ſmaller *Lenſes*, where this Limitation of the *Aperture* is neceſſary, the Rule is, that the Diameters of thoſe *Apertures* ſhould be in the ſame proportion with the *focal* Diſtances of the reſpective *Lenſes*, in order to have the Object ſeen by both equally diſtinct. But the *Light* or *Brightneſs* will be in a duplicate Proportion of thoſe *focal* Diſtances; ſo that the more convex the *Lens* is, the greater indeed, but then the more obſcurely will every thing be ſeen.

[*Fig.* 14.] Let P be a ſmall *Lens*, whoſe Axis is T B F, P D the Semidiameter of the Aperture, which Experience teaches to be the greateſt that can be admitted, and that leſs than the Pupil of the Eye, F the extreme *Focus* of *red Rays* (which are leaſt refrangible) proceeding parallel to the Axis, in which Point ſuppoſe the Object to be placed, and B the *Focus* of *violet-coloured Rays, which are moſt refrangible.* The ſame things being ſuppoſed in a ſmaller *Lens p*, the Semidiameter of whoſe *Aperture pd* is to the *focal* Diſtance *pf* in the ſame Proportion as in the greater; I ſay the Object will be ſeen equally diſtinct in both.

For ſince the Ray E D, parallel to the Axis falling upon the *Lens* P, is refracted unequally, and divided into its extreme Colours by the Angle F D B, ſo that the extreme Colour Red paſſes to F, and the extreme Violet-colour to B; it will happen on the contrary, that a Ray FD proceeding from the Object will be divided into its extreme Colours by an Angle EDK, equal to FDB. Therefore in both Caſes FDB is the *Angle of Aberration*, upon which depends the Aberration of the Rays in the bottom of the Eye, as ſhall be ſhewn when we come to ſpeak of *Teleſcopes*. But ſince from the Nature of this *Abberration*, PF is to FB as *pf* to *fb*; and alſo by Conſtruction PD is to PF as *pd* to *pf*; it follows that the Angles, as well PFD, *pfd*, as PBD, *pbd*, are equal. Wherefore the Difference of the Antecedents, PFD, PBD, is equal to the Difference of the Conſequences, *pfd*, *pbd*; that is, the Angle FDB is equal to the Angle *fdb*, and conſequently the *Aberrations* in the bottom of the Eye are in both Caſes equal, and by that means Viſion equally diſtinct.

Moreover, becauſe the Angles PFD, *pfd* are equal, it is plain that the ſame Quantity of Rays in both Caſes proceeds from the ſame Points of the Object F and f, or any others, upon the *Lenſes*, and from thence to the Eye. But the Breadth of the Object in the bottom of the Eye is in the ſmaller *Lens* ſo much greater as PF is greater than *pf*, as has been before demonſtrated; and the apparent Surfaces are in a duplicate Proportion of thoſe Breadths.

There-

Therefore the ſame Quantity of lucid Rays expended towards illuſtrating each Surface, will make that which is leaſt the cleareſt by ſo much as the other Surface is greater; that is, in a duplicate Proportion of PF to *pf*; which was the laſt thing to be demonſtrated.

Since therefore the ſame Perfection of Viſion which is to be found in larger *Lenſes* cannot be had in more convex *Lenſes* without diminiſhing the Brightneſs of the Object at the ſame time; it follows that we cannot proceed in magnifying as much as we pleaſe, unleſs a greater Light be borrowed ſomewhere elſe to illuſtrate the Object. Nor will this be of any great Benefit, becauſe the Latitude at the Pupil of the Eye, or the little Cylinder of Rays flowing from every Point of the Object, and which has here the ſame Latitude with the Aperture, cannot be contracted farther than the fifth or ſixth part of a *Line*; ſo that even this limits the Efficacy of theſe little *Lenſes*.

The Effects of more compounded *Microſcopes* will eaſily be accounted for after the ſame manner. And indeed a full Conſideration of *Prop.* XXI. and XXII. is ſufficient for explaining the Effects of all ſorts of compound *Microſcopes*.

Of

Of TELESCOPES.

PROP. III.

A Teleſcope *made by a* convex *and* concave Lens *repreſents vaſtly diſtant Objects diſtinct and erect; and magnifies them according to the Proportion of the* focal *Diſtance of the* convex Lens *to the* focal *Diſtance of the* concave Lens.

Fig. 15. Let AO be the common Axis of both *Lenſes*, and A the extreme *convex Lens*, whoſe *Focus* of parallel Rays proceeding from the vaſtly diſtant *Object* is ſuppoſed to be at O. Let D be the *concave Lens*, which is ſo placed between the *Lens* A and its *Focus* O, that the ſame Point O may alſo be the *Focus* of the *concave Lens*, where Rays falling parallel from the Side of O would be collected. And firſt, ſuppoſe the Eye of the Spectator placed next to this *Lens*.

The Rays then proceeding parallel from each Point of the *vaſtly diſtant Object*, and falling upon the *Lens*, thoſe which proceed from that Point of the *Object*, which is in the Axis produced, would be collected at the Point O; but they are again made parallel by means of the *Lens* D. We would have the Rays fall parallel upon the Eye, that the Teleſcope may be fitted for thoſe who have good Eyes; for we ſhall ſpeak afterwards of ſhort-ſighted Eyes. In like manner the Rays proceeding from thoſe Points of the *vaſtly diſtant Object* which are out of the Axis would be all collected at reſpective Points near O; but theſe alſo by *Refraction* at the *Lens* D are again made parallel, though ſomething oblique to the Axis A D, which Rays, to avoid Confuſion, are not expreſſed in the Figure. Therefore the Rays which proceed from the *vaſtly diſtant Object* being made to fall parallel upon the Eye, will make *diſtinct Viſion*; and ſince thoſe Rays that proceed from the *Object* go on to meet the Eye in the ſame Order, 'tis plain the apparent Propoſition of the *Object* will be the ſame with the true, or the *Object* will be *erect*.

Fig. 16. The *Lenſes* A C and D, and the Point O being placed as before, find by *Prop.* XVI. the Point P, to which Rays tending, will, by *Rafraction* at the *Lens* A C, be collected at D the Center of the *Lens* D: Which Point is alſo found, by making D P a third proportional to D O, D A, and taking it on the ſame Side with D O. Suppoſe the Ray E C P to be one of thoſe which proceed from the extreme right Side of the *vaſtly diſtant Object*, which imagine to be the Moon, and its Center to be placed in the Axis D A produced. It is plain that this Ray will come to the Eye in the right Line C D F, becauſe it paſſes thro' the Center of the *Lens* D, whoſe middle Thickneſs may be neglected as inconſiderable, and its two Surfaces in that Place looked upon as parallel. But we have ſhewn before, that all the Rays proceeding from each Point of the Moon will, by means of ſuch a Teleſcope, fall parallel upon the Eye. Wherefore the Eye will receive all the Rays from that Point E of the Moon in ſuch manner, as that they ſhall be parallel to the Ray C D F; and conſequently will ſee that Point of the Moon in the Place to which the right Line D C tends; which tending to the ſame Side of the Axis, on which that Point of the Moon is ſituated, from whence the Rays proceeded, 'tis plain the Object will appear *erect.* Moreover the Angle A D C determines the Semidiameter of the Moon, as encreaſed by the Teleſcope. But the Angle C P A is that which determines its Semidiameter, as ſeen by the naked Eye; becauſe we before ſuppoſed the Ray E C P to proceed from the extreme right Side of the Moon, and the Ray H A P from its Center: For though the Point P is beyond the Eye, and the Eye ſees from the Point O, yet the Moon being an Object vaſtly diſtant, will appear under the ſame Angle to the naked Eye, whether it be viewed from the Point P or O. Therefore the Moon will appear magnified, according to the Proportion of the Angle A D C to A P C, which Proportion may here be looked upon as the ſame with that of P A to D A. But becauſe by Conſtruction D O is to D A as D A to D P, by inverting and compounding the Proportion, A O will be to O D as P A to A D. Wherefore the apparent Magnitude will be to the true as A O to O D. *Q. E. D.*

It appears from hence, that the apparent Magnitude is the ſame, in whatſoever Place behind the *Lens* D the Eye is ſituated.

Fig. 17. Let the *Lenſes* A C and D be placed as before, and let A Q be taken in their Axis produced equal to A O. And out of thoſe Rays which proceed from a Point of the right Side of the Moon, let us conſider the Ray R Q C paſſing through the Point Q (for ſome one will paſs through it) and meeting with the *Lens* A C in C. It will afterwards become parallel to the Axis A D, and when refracted again at the concave *Lens*, will diverge, as if it came from the Point L, and will tend to the Eye in the right Line L I F, ſo as that the Diſtance L D may be equal to D O; becauſe L is in that Caſe the *Focus* of parallel Rays falling upon the *Lens* D.

The Proportion of the encreaſed Magnitude is hence eaſily collected. For becauſe the Rays proceeding from the right Extreme of the Moon, after having paſſed both *Lenſes*, are parallel, and come ſo to the Pupil G F, and conſequently they become parallel to the Ray L I F, which we know to be one of them; that Point of the Moon will appear in the right Line I L, and conſequently the Semidiameter of the Moon will be comprehended in the Angle I L D. But the Angle in which the Semidiameter would appear to the naked Eye, either from D or from Q, is R Q H, or C Q A. Therefore the Proportion of the encreaſed Magnitude is the ſame with that of the Angle D L I to A Q C; that is (becauſe of A C, D I equal) as A Q to L D. But A Q is equal to A O, and L D is equal to D O. Therefore the Proportion of the apparent Magnitude to the true is as A O to O D. *Q. E. D.*

To determine what will be the Amplitude of the viſual Angle, *or of the Space which is repreſented at one View by a* Teleſcope *conſiſting of a* convex *and* concave Lens.

Fig. 18. The Amplitude of the *viſual Angle* in theſe *Teleſcopes* depends chiefly upon the Magnitude of the Pupil of the Eye, which is confirmed by Experiment. For if applying your Eye to the Teleſcope, you firſt ſhut it, that the Pupil may be dilated as it uſually is in the Dark, and then open it on a ſudden, at firſt View you will diſcern Objects in a larger Orb than a little while afterwards, the Orb being preſently contracted as ſoon as the Pupil is contracted by the Brightneſs of the Light. But if you place a Plate perforated with a ſmall Hole before the Eye, you will diſcern every Object in a leſſer Orb.

If you make the Hole extremely ſmall, the lucid Orb will not be contracted in proportion to the Smallneſs of the Hole, but its Amplitude will then be limited by the Aperture of the convex *Lens*, and conſequently will not be diminiſhed beyond a certain Degree, except the convex *Lens* be alſo more contracted. The Reaſon of which is very eaſy to be explained. For if E F be the convex *Lens*, and B the concave, to which the Pupil of the Eye applied has firſt the Magnitude C D; draw from the oppoſite Points C, D in the Circumference of the Pupil, through the Center of the *Lens* A, the right Lines C A H, D A G. Theſe will determine the *viſual Angle*, under which that Part of any Object which is ſeen at one View is comprehended: Becauſe Rays coming from the Points G, H through the Center of the *Lens* A, penetrate without Inflexion to C and D; therefore

fore that Part of the Object which is comprehended within the Angle G A H cannot but ſend Rays to the Eye, even though the Pupil were a little narrower than D B C. For drawing G A K ſo as to make A K equal to A O, and joining E K; if ſo be E K falls upon the Pupil, the Object comprehended under the Angle G A H will be diſcerned, but the extreme Points towards which the right Lines A G, A H tend will be ſeen but obſcurely, becauſe only a ſmall Part of the Rays, which they caſt upon the *Lens* E F, enter the Pupil. And hence it happens, that how much ſoever the Aperture of the *Lens* E F is contracted, the Amplitude of the *viſual Angle* is nevertheleſs not at all, or extremely little diminiſhed, ſo the Orb of the Pupil be not contracted. But this Breadth of the Pupil being diminiſhed, and reduced as it were to a Point, the Amplitude of the *viſual Angle* is the ſame with that of the Angle E P F; E F being ſuppoſed the Aperture of the convex *Lens*, and the Point P found by *Prop.* XVI. or by making B O (the Diſtance of the concave *Lens* from the *Focus* of the convex) B A and B P in a continual Proportion. For no Rays tranſmitted through the *Lens* A can arrive at the Point of the Eye B, but ſuch as before they fall upon that *Lens* tend towards the Point

P. The greateſt Angle E P F, of which Rays, is determined by the Aperture of the *Lens* A.

This is the *Teleſcope* which was firſt ſound out by *Galilæus*, and ſtill retains his Name; and is the ſame with a common Proſpective Glaſs.

Prop. IV.

A Teleſcope *made of two* convex Lenſes *repreſents vaſtly diſtant Objects diſtinct, but inverted; and magnifies them according to the Proportion of the* focal *Diſtance of the exterior or* object Lens, *to the* focal *Diſtance of the interior or* ocular Lens.

[*Fig.* 19, 20.] Let A C be the exterior convex *Lens*, D the interior, A D the common Axis of both, and O the *Focus* of the *Lens* A C. Let the other Convex D be ſo placed, that the ſame Point O may be alſo its *Focus*, or the Point of Concourſe of parallel Rays coming from the Side of G, where the Eye is ſuppoſed. We are to ſhew that all this being ſuppoſed, vaſtly diſtant Objects will be ſeen diſtinct, and inverted and magnified, according to the Proportion of AO to OD.

And here we muſt make uſe of two ſeveral Figures, as in the *preceding Propoſition*; in the firſt of which the Rays coming parallel to the Axis H A, are by the Refraction of the Lens A C collected at its *Focus* O, and from thence tending farther to the *Lens* D, are by it again made parallel to the Axis A D, and ſo come to the Eye placed at G. And as in the *preceding Prop.* we muſt again conſider this Compoſition of parallel Rays, as coming from a ſingle Point of the vaſtly diſtant Object, which is placed in the Axis H A D, as ſuppoſe from the Center of the Moon; and the like parallel Rays coming from every other Point of the *Object* upon the *Lens* A C, as ſuppoſe from the extreme right Side of the Moon, which are inclined to the former, and being thereby refracted, are collected in a Point of the Axis near O; where interſecting themſelves, and proceeding to the *Lens* D, after they have gone through it, they are again made parallel, (that is, only among themſelves reſpectively) and ſo arrive at the Eye. Whence 'tis plain, Viſion will be made diſtinct.

The other Figure ſhews the inverted Situation, and the Proportion of the encreaſed Magnitude of the *Object*. Where the convex *Lens* A C and D, and their common *Focus* O, being placed as before, and moreover, as in the ſecond Demonſtration of the *preceding Prop.* the Diſtance A Q being made equal to A O, the remaining Part of the Demonſtration will proceed much after the ſame manner. For if, out of the Rays which proceed from a Point in the extreme right Side of the Moon, we chooſe one R Q C paſſing through the Point Q; that, after Refraction at the *Lens* A C, will paſs in C I parallel to A D, and being again refracted by the *Lens* D, will tend along the right Line I F L to the Point L, taken in ſuch manner that the Diſtance E L is equal to D O. But becauſe the Rays from the extreme right Side of the Moon, after Refraction at both *Lenſes*, arrive parallel at the Eye, as has been ſaid before, and I F L is one of them; it follows that they will all fall parallel to I F L upon the Eye, and that Point of the Moon will be ſeen in a Place, according to the right Line F I: Which ſince it tends to the oppoſite Side to that from whence thoſe Rays came, 'tis plain that the Situation of the Moon will appear inverted, ſo that the right Side will be changed to the left, and the upper Parts to the lower. Moreover, ſince the Center of the Moon will be ſeen in the right Line D A, I L D will be the apparent Angle of the Semidiameter of the Moon. But to the naked Eye that Semidiameter is comprehended under the Angle HQR, or AQC. Therefore the *Ratio* of the apparent Magnitude to the true is as the Angle DLI to AQC, that is, as AQ to DL; becauſe AC, DI are equal, that is, as AO to OD. *Q. E. D.*

And here likewiſe it appears, that it ſignifies nothing to the apparent Magnitude, whereſoever the Eye is placed behind the *Lens* D. But that it may comprehend moſt at one View, it is convenient it ſhould be placed at or near the Point L; becauſe it appears, that although the Breadth of the Pupil be ever ſo little, yet the whole *Lens* D, while it does not exceed the Aperture of the *Lens* AC (for it is uſually confined within this Meaſure) will be ſeen full of the Object.

This is the *Teleſcope* moſt commonly uſed to look at celeſtial Bodies.

PROP. V.

TO *explain the Conſtruction of a* Teleſcope *compounded of four* Convexes, *by means of which Objects are ſeen* erect *and very* amply.

[*Fig.* 21, 22.] *Teleſcopes* made of *two Convexes*, becauſe of their inverging the Poſition of the Object, are ſeldom uſed, except in obſerving the Stars, the Poſition of which is not regarded. The Proportion in which this Sort magnifies the Object has already been demonſtrated. But if we would have theſe Images again made *erect*, and at the ſame time a great Share of them be repreſented to the Eye at one View very *amply*, we muſt uſe 3, 4, 5, or more *Lenſes*. Which however are not to be multiplied without Cauſe, becauſe the Matter of each of them and the Reflexion of their ſeveral Surfaces divert Part of the Rays. But we cannot obtain the deſired Effect perfectly, with fewer than 4 *Lenſes*. For although in the ſame Length of the *Teleſcope* both an *erect* Situation and the ſame Degree of Magnifying, and an equal Share of the Object may be had as well with 3 as 4 *Lenſes*; yet the Compoſition of 3 *Lenſes* is much more inconvenient than that of 4, becauſe in that the two ocular *Lenſes*, or at leaſt that which is next the Eye, muſt be made of larger Segments of a Sphere, with reſpect to its Diameter, or to the *focal* Diſtance, if the ſame Magnitude of the viſual Angle be required. And hence the Objects come to be coloured, and right Lines, at the Margins of the Aperture, appear curve.

Therefore we muſt make our Teleſcope of 4 *Lenſes*; which is done after the following manner.

The exterior or object *Lens* is A, whoſe *focal* Diſtance is A B; and in the ſame Axis are placed three ocular *Lenſes*, C, D, and E, all equal to one another, the inmoſt of which is placed beyond the *Focus* B, by its *focal* Diſtance B C, and the next D is placed beyond C, by twice that Diſtance, B C, and the laſt as far from D as that was from C; and laſtly, the Eye muſt be placed beyond this laſt by the Diſtance B C.

There is here again Occaſion for two Figures; in the firſt of which are repreſented Rays proceeding from a ſingle Point of the vaſtly diſtant Object: Which ('tis plain to any who underſtand what has gone before) firſt fall as it were parallel upon the *Lens* A, and are by it collected at its *Focus* B, and thence diverging fall upon the *Lens* C, which makes them again parallel, and throws them upon the *Lens* D, which collects them at its *Focus* H, the middle Point of the Diſtance D E; from whence proceeding on to the *Lens* E, they are by it made a third time parallel, and being received ſo by the Eye F, they make diſtinct Viſion, by being collected at its *Focus* which is in the bottom of the Eye.

The other Figure conſiders the *Proportion* of *Magnifying*; which is, *That which*

AB, *the* focal *Diſtance of the object* Lens, *bears to* BC, *the* focal *Diſtance of one of the ocular* Lenſes: And demonſtrates likewiſe the Amplitude of the viſual Angle. For the Apertures of the three ocular *Lenſes* being ſuppoſed equal, which muſt not exceed the Aperture of the object *Lens* A, draw MQ, NR parallel to the common Axis, and comprehending the Diameters of the Apertures of the *Lenſes* E and D. And alſo KO, LP parallel to the ſame Axis, and comprehending KL, the Aperture of the *Lens* C; and taking AG equal to AB, draw the right Lines OGU, PGT interſecting one another in G. Now it is evident the Latitude of the Object, which if ſeen by the naked Eye from the Point G, and conſequently from F alſo, the Diſtance of the Object being as it were infinite, would appear comprehended in the Angle TGV; if ſeen through the Teleſcope, would appear comprehended in the Angle MFN: And conſequently the Proportion of the apparent Magnitude to the true is as the Angle MFN to the Angle TGV, or PGO; that is, PO and MN being equal, as the Diſtance AG to the Diſtance EF; that is, as AB, the *focal* Diſtance of the object *Lens*, to BC, the *focal* Diſtance of one of the ocular *Lenſes*. *Q. E. D.*

It appears moreover, that the viſual Angle MFN comprehends the ſame Latitude of the Object with a Teleſcope made of two *Lenſes* only, A and C; for that Share of the Object which is comprehended in the Angle T G V, would be ſeen through that Teleſcope in the Angle K S L equal to the Angle M F N.

This incomparable Compoſition of *Lenſes* was found out by I know not whom at *Rome*, and may be much improved by placing an *Annulus* or Ring either at H, the common *Focus* of the *Lenſes* D and E, or at B the common *Focus* of the *Lenſes* A and C; which is eſpecially of very great Uſe in meaſuring the Diameters of Planets. For this *Annulus* does therefore exactly circumſcribe the Circle of the apparent Images, becauſe it cuts off thoſe irregular Rays which are not collected near enough to B or H, and conſequently are not by means of the ſucceeding *Lenſes* ſent parallel to the Eye, which diſtinct Viſion requires: And the Colours likewiſe near the Margins are by this Contrivance taken away, which without it are not well to be avoided.

It may ſeem a little ſtrange, that the Colours of the Iris ariſe no more in this Teleſcope, by the Refraction of ſo many ocular Lenſes, than in that where there is but one; but to any one that ſhall conſider it, the Reaſon will be very obvious. For the Lens Q R corrects and takes away thoſe Colours,

which

which the Lens K L produced, their ſpherical Surfaces being equal by Conſtruction.

Of the APERTURE *of the* LENSES.

Since the Proportion of magnifying in *Teleſcopes* made of two Lenſes has been ſhewn to be that which the *focal* Diſtance of the object *Lens* bears to the *focal* Diſtance of the ocular *Lens*, it may be thought perhaps, that however ſhort the *Teleſcope* be, the Object may be magnified in any aſſigned Proportion. But there are two Cauſes which make this impoſſible; one is, that the *Aperture* of the object *Lens* remaining the ſame, the more we magnify the Object by uſing a leſs convex ocular *Lens*, the more obſcure we make them appear. The other is, that it repreſents them leſs diſtinct. And if we expect a Remedy by encreaſing the *Aperture*, the Confuſion will be the more encreaſed. What belongs to the *Brightneſs* or *Obſcurity*, will be eaſily underſtood by attentively conſidering the Image of any Object painted upon the bottom of the Eye; which the greater it is made, whether by means of the Refraction of *Lenſes*, or only by approaching nearer, in ſo much greater Plenty muſt the Rays from every Point be received within the Eye, in order that the ſame *Brightneſs* may ſtill remain. For if looking at an Object with the naked Eye, you approach to it twice as near, the Image at the bottom of the Eye will be twice greater in Diameter, and four times in Area. But four times more Rays do alſo, from every Point of it, enter the Pupil of the Eye; becauſe the Angle made by the Cone of Rays becomes twice as large: And therefore it is that the ſame *Brightneſs* of the Image is perceived at both Diſtances, which is the Contrivance of Nature. But if a *Teleſcope* were to be made which ſhould magnify the Diameter of any Object ten times, and repreſent it as bright as when it is looked at with the naked Eye, the Diameter of the Aperture of the Object Lens ought to be ten times greater than the Diameter of the Pupil, although no Part of the Rays were intercepted by the Reflexion of the Surfaces of each *Lens*, or by the Colour of the Glaſs. For by this means, when the Surface of the Object is magnified an hundred times, we have alſo an hundred times more Light than was received by the naked Pupil.

But a much leſs Meaſure of *Brightneſs* ſuffices for *Teleſcopes*; for thoſe which we uſe in the Day-time are not too obſcure, if they have but $\frac{1}{6}$ or $\frac{1}{7}$ of that *Brightneſs* which is uſually perceived by the naked Eye. But thoſe longer ones with which we obſerve the Moon and the Planets, require not above half this laſt *Brightneſs*, becauſe the Eye is moved with a much leſs Brightneſs in the Night than in the Day. So that in a *Teleſcope* 30 Feet long, which magnifies the Diameters of the Planets 109 times, and would conſequently require the Diameter of the Aperture of the object *Lens* 109 times greater than the Diameter of the Pupil, that is, of about 11 Inches, if we ſuppoſe the Diameter of the Pupil to be $\frac{1}{10}$ of an Inch; we find that an Aperture of 3 Inches in Diameter ſuffices, which admits leſs than $\frac{1}{13}$ of that *Brightneſs* which would be admitted by an Aperture of 11 Inches.

The Proportions between the *focal* Diſtances of the object *Lens* (which is likewiſe the Length of the Teleſcope) the *Aperture* of the ſame object *Lens*, the *focal* Diſtance of the ocular *Lens*, and the apparent magnified Diameter of the Object, for *Teleſcopes* from the Length of 1 Rhinland Foot to 100, are expreſſed in the Table following.

TABLE

The focal Diſtance of the object *Lens*, or the Length of the *Teleſcope*.	The Diameter of the Aperture of the object *Lens*.	The focal Diſtance of the ocular *Lens*.	The Proportion of Magnifying, conſidered as to Diameter.
Rhinland Feet.	*Inches, & Decimals.*	*Inches, & Decimals*	
1.	0,55.	0,61.	20.
2.	0,77.	0,85.	28.
3.	0,95.	1,05.	34.
4.	1,09.	1,20.	40.
5.	1,23.	1,35.	44.
6.	1,34.	1,47.	49.
7.	1,45.	1,60.	53.
8.	1,55.	1,71.	56.
9.	1,64.	1,80.	60.
10.	1,73.	1,90.	63.
13.	1,97.	2,17.	72.
15.	2,12.	2,33.	77.
20.	2,45.	2,70.	89.
25.	2,74.	3,01.	100.
30:	3,00.	3,30.	109.
35.	3,24.	3,56.	118.
40.	3,46.	3,81.	126.
45.	3,67.	4,04.	133.
50.	3,87.	4,26.	141.
55.	4,06.	4,47.	148.
60.	4,24.	4,66.	154.
65.	4,42.	4,86.	161.
70.	4,58.	5,04.	166.
75.	4,74.	5,21.	172.
80.	4,90.	5,39.	178.
85.	5,05.	5,56.	183.
90.	5,20.	5,72.	189.
95.	5,34.	5,87.	194.
100.	5,48.	6,03.	199.

PROP. VI.

TO explain the Manner of fitting a Teleſcope *for obſerving* Eclipſes *of the* Sun, *and diſcovering the Spots in its Surface, and to determine how great its Image will be repreſented.*

[*Fig.* 23, 25.] A *Teleſcope* is found to be of great Uſe in obſerving *Eclipſes* of the Sun, and alſo in diſcovering the Spots which are ſaid to be in its Surface; by receiving the Image formed by both its *Lenſes* upon a white Plane, from which the Light is every other way excluded. In order to explain which Invention we muſt firſt demonſtrate the Poſition of the *Lenſes* which is neceſſary to form the Image of the Sun, as clear and diſtinct as may be.

Let A B be the convex *Lens* next the Sun, whoſe *Focus* is E. The other is D, either concave or convex; for either of theſe Sorts of *Teleſcopes* will do the Buſineſs: Though a Teleſcope of two *Convexes* is the moſt convenient, becauſe we make it repreſent the Images erect, while by the other Sort we invert them. Let the Point K be the *Focus* of the *Lens* D, where the Rays coming from the Side of H parallel, are after Refraction by it collected, and in H ſuppoſe the white Plane placed in order to receive the Image of the Sun. Which that it may appear diſtinct and nicely terminated, 'tis neceſſary that the Rays which proceed from any one Point of the Sun, and which fall parallel upon the *Lens* A B, ſhould again be collected in one Point upon the Plane. Wherefore the Diſtance between the *Lenſs* A B and D ought to be ſomething greater than in the common Diſpoſition of the *Teleſcope*, or than when it is fitted for a good Sight; and the Poſition of the *Lens* B ought to be ſuch, that the Rays which would otherwiſe tend to the *Focus* E of the *Lens* A B, may be diverted and brought to H; which may be done by *Prop.* XVI. or by taking E K, E D, E H in a continual Proportion. But in the common Diſpoſition of the Teleſcope, the *Focus* K is required to coincide with the *Focus* E, as has been ſhewn above. So that here the Diſtance of the *Lenſes* is encreaſed by the Space E K, which will always be ſo much leſs, as the Diſtance E H is increaſed. For the Diſtance D K, which is the given *focal* Diſtance of the *Lens* D, is divided in ſuch manner in E, that H E is to E D as the ſame E D to E K.

[*Fig.* 24, 26.] How great the Diameter of the Image of the Sun will appear upon the Plane H may thus be determined. Draw from the Center of the *Lens* A B to the *Lens* D the right Lines B P, B Q, comprehending an Angle equal to that, under which the Sun's Diameter appears without a *Teleſcope*; and make BC a third Proportional to BK, BD, and join CP, CQ; which will conſequently give G the *Focus* of Rays diverging from B after Refraction at the *Lens* D; which produce till they meet the Plane placed at H in the Points L, M. I ſay, LM will be the Diameter of the Sun's Image repreſented upon the Plane LHM. For produce PB, QB towards O and N. Therefore ſince from the extreme Point of the real Diameter of the Sun on the right Hand Side, Rays are ſent upon the whole *Lens* AB, which are all to be looked upon as parallel among themſelves, and to the right Line OB, one of them will proceed along the right Line OB, and penetrating the *Lens*, gon on in the right Line BP; becauſe B is the Center of the *Lens*, whoſe Thickneſs is here neglected. For the ſame Reaſon one of the parallel Rays from the left Extreme of the Sun's Diameter will proceed along the right Line NBQ. But moreover, both will be refracted in ſuch manner by the *Lens* D, that diverging from their *Focus* C, they will proceed along the right Lines PL, QM, which are the right Lines CP, CQ produced. Therefore 'tis plain the Point in the right Hand Extreme of the Sun will be repreſented at L, and the oppoſite Point in the left Extreme at M.

For ſince the Image of the Sun is required diſtinct, it is neceſſary that where one Ray proceeding from any Point of it falls upon the Plane, all the reſt which proceed from the ſame Point ſhould be collected there alſo. Therefore the Diameter of the Image is L M: And by a *Teleſcope* made of a Convex and Concave, the Image is inverted; and by one made of two Convexes it is repreſented erect. *Q. E. D.*

But it muſt be obſerved, that the greater L M the Image of the Sun is, the *Lens*'s A B and D remaining the ſame, the leſs clear and diſtinct will it be: For if all the Rays deſcending from the Sun upon the *Lens* A B, ſhould poſſeſs a Space in the Place L H M equal to the Breadth of the *Lens* A B; that is, if they were to form the Image of the Sun equal to the Aperture of the *Lens* A B, this Image would be as clear as if the Plane were enlightened by the Sun without the Interpoſition of *Lens*'*s*. No reſpect being had to thoſe Rays which the *Lens*'*s* reflect, or by reaſon of their imperfect Tranſparency do not tranſmit, which perhaps occaſions a Loſs of above half the whole Number of Rays. But if the Sun's Image be made larger, which is neceſſary to be done in Obſervations of this nature, it will then be ſo much the more obſcure. But Experience is the beſt Judge to determine in what Magnitude it will be moſt convenient to repreſent the Sun's Image in theſe Obſervations; by trying firſt one, and then another Diſtance of the *Plane* from the *Teleſcope*. Where it is to be obſerved, that as we encreaſe this Diſtance, the Diſtance between the *Lens's* A B and D ought to be a little diminiſhed, in order to preſerve the Diſtinctneſs of the Image; the Reaſon of which has been already given.

W. BROWNE.

APPENDIX

To this SECOND EDITION,

By J. T. DESAGULIERS, LL.D. and F.R.S.

THE firſt Thought we find in Print concerning a *Reflecting Teleſcope* is that of Dr. JAMES GREGORY, our Author's Uncle; who, in his *Optica promota* (Pag. 93 and 94) propoſes a Catadioptrical Teleſcope (as being far preferable to Two other Sorts of Teleſcopes which he has been deſcribing *) with no other View, but to make Teleſcopes ſhorter and more handy: For he and the reſt of the World were at that time wholly unacquainted with the different Refrangibility of the Rays of Light; and conſequently ignorant of the Errors occaſioned in Refracting Teleſcopes by that Property of the Rays. Sir ISAAC NEWTON, the Diſcoverer of that wonderful *Phænomenon* in Light, being then alſo unacquainted with it; not having made any of his Experiments with the *Priſm* till the beginning of the Year 1666.

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* See his Words at the End of this *Appendix*, No. I.

Dr. J. GREGORY never brought this Teleſcope to any Degree of Perfection; but, owning his Want of Skill in Mechanics, only propoſes it for others to execute. He had indeed an Object-*Speculum* of Metal ground to a Segment of a Sphere, and a little concave, as well as a little convex Speculum, ground by *Rives* and *Cox* (Optic-Glaſs-Grinders, famous in thoſe Days) But becauſe the Object Metal was not well poliſhed, he only made ſome imperfect Trials, not ſo much as fitting the *Specula* and Eye-Glaſs into a Tube; and being diſcouraged, as much becauſe he could not have the Object Metal ground into a parabolic Concave, as becauſe that which he tried was not well poliſhed, he gave over the Thoughts of bringing ſuch Teleſcopes into uſe. (See his Letter to Mr. *Collins*, written from Saint *Andrews*, *Sept.* 23, 1672. N°. 4.)

But Mr. J. HADLEY, a few Years ago, after having made the *Newtonian* Reflecting Teleſcopes with good Succeſs (the Deſcription of which may be ſeen in N°. 376 of the *Philoſophical Tranſactions*) improving all Dr. J. GREGORY's Hints, did alſo bring this Teleſcope to Perfection by the Work of his own Hands; and has ſince taught his Majeſty's Optician Mr. EDWARD SCARLET, and his Son, to make both the Sorts; which they do ſo well, that I have not yet known them exceeded in theſe Inſtruments by the Performance of any other Optic-Glaſs-Grinder.

To return to the Hiſtory of the Reflecting Teleſcopes.

Sir ISAAC NEWTON, in the Year *1666*, applied himſelf to grind Optic Glaſſes of other Figures than ſpherical (ſuppoſing, with all other Perſons who had hitherto applied themſelves to the Study of Optics, that Teleſcopes might be very much improved, by making uſe of Glaſſes ground to the Figure of ſome other conic Section; ſuch as the parabolical, elliptical, or hyperbolical) but having made ſome Experiments with Priſms, whereby he diſcovered the different Refrangibility of the Rays of Light, he found that the Errors in Teleſcopes ariſing from that Cauſe was ſome hundreds of times greater than thoſe which were occaſioned by the ſpherical Figure of the Glaſſes, which did not collect the Rays into one Point, where Glaſſes of the Figure of a conic Section would do it if Light was uniform. This made him take Reflexions into Conſideration; and finding them regular, ſo that the Angle of Reflexion of all ſorts of Rays was equal to their Angle of Incidence, he underſtood that by their Mediation Optic Inſtruments might be brought to any Degree of Perfection imaginable, provided a *reflecting* Subſtance could be found, which would poliſh as finely as Glaſs, and *reflect* as much Light as Glaſs *tranſmits*; and the Art of communicating to it a *parabolic* Figure be alſo attained. But he thought the Difficulties very great, and almoſt inſuperable, when he further conſidered, that every Irregularity in a reflecting Superficies makes the Rays ſtray five or ſix times more out of their due Courſe, than the like Irregularities in a refracting one: So that a much greater Curioſity would be here requiſite, than in figuring Glaſſes for Refraction.

Being then forced from *Cambridge* by the intervening Plague, he did not proceed any farther till two Years after.* Then, having conſidered what Dr. J. GREGORY propoſed in his *Optica promota* concerning a *catadioptric* Teleſcope, with an Hole in the midſt of the Object Metal to tranſmit the Light to an Eye-Glaſs placed behind it, he thought the Diſadvantages would be ſo great, that he reſolved, before he attempted any thing in the Practice, to alter Dr. GREGORY's Deſign, and place the Eye-Glaſs at the Side of the Tube, rather than the Middle. † He then made two little Reflecting Teleſcopes with an Object Metal ſpherically concave; one of which he deſcribes in the *Philoſophical Tranſactions* (N°. 80) and the other, which was better than the firſt, he ſent to the Royal Society.

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* *Phil. Tranſ.* No. 80. † *ib.* No. 83.

This laſt is deſcribed in a Letter to a Friend (ſee the *Originals* hereto ſubjoined, N°. 2) but more fully in the ſaid *Tranſactions*, N°. 81; from which we here tranſcribe the Deſcription, with Mr. HUYGENS's Remarks, and Sir ISAAC's Reply to them; as alſo a farther Account of the ſame Teleſcope, with a Table of *Apertures* and *Charges*, from the 82d *Tranſaction.*

An Account of a new Catadioptrical Teleſcope *invented by Mr.* NEWTON, Fellow *of the* Royal Society, *and* Profeſſor *of the* Mathematics *in the Univerſity of* Cambridge.

" This excellent Mathematician having " given us, in the *Tranſactions* of *February* laſt, an Account of the Cauſe which " induced him to think of Reflecting Teleſcopes, inſtead of refracting ones, hath " thereupon preſented the curious World " with an Eſſay of what may be performed " by ſuch Teleſcopes; by which it is found " that Teleſcopical Tubes may be conſiderably ſhortened, without Prejudice to " their magnifying Effect.

" This new Inſtrument is compoſed of " two metalline Speculums; the one concave, inſtead of an Object Glaſs, the other " plane; and alſo of a ſmall plano-convex " Eye-Glaſs.

“ By *Fig.* 2. of *Tab.* IV. the Structure
“ of it may be eaſily imagined; *viz.* That
“ the Tube of this Teleſcope is open at the
“ End which reſpects the Object; that the
“ other End is cloſe, where the ſaid Con-
“ cave is laid; and that near the open End
“ there is a flat oval *Speculum*, made as
“ ſmall as may be, the leſs to obſtruct the
“ Entrance of the Rays of Light, and in-
“ clined towards the upper Part of the
“ Tube, where is a little Hole furniſhed
“ with the ſaid Eye-Glaſs: So that the
“ Rays coming from the Object, do firſt
“ fall on the Concave placed at the bottom
“ of the Tube; and are thence reflected
“ toward the other End of it, where they
“ meet with the flat *Speculum*, obliquely
“ poſited; by the Reflexion of which they
“ are directed to the little plano-convex
“ Glaſs, and ſo to the Spectator's Eye,
“ who looking downwards, ſees the Object
“ which the Teleſcope is turned to.

“ To underſtand this more diſtinctly and
“ fully, the Reader may pleaſe to look
“ upon the ſaid Figure; in which

“ A B is the concave *Speculum*, of which
“ the *Radius* or Semidiameter is $12\frac{2}{3}$ or
“ 13 Inches.

“ C D, another metalline *Speculum*,
“ whoſe Surface is flat, and the Circumfe-
“ rence oval.

“ G D,

" G D, an Iron Wire, holding a Ring
" of Braſs, in which the Speculum C D is
" fixed.

" F, a ſmall Eye-Glaſs, flat above, and
" convex below, of the twelfth part of an
" Inch Radius, if not leſs; foraſmuch as
" the Metal collects the Sun's Rays at $6\frac{1}{3}$
" Inches Diſtance, and the Eye-Glaſs at
" leſs than $\frac{1}{6}$ of an Inch Diſtance from its
" Vertex: Beſides that the Author (as he
" informs us) knew their Dimenſions by the
" Tools to (or in) which they were ground,
" and particularly meaſuring the Diameter
" of the hemi-ſpherical Concave, in which
" the Eye-Glaſs was wrought, found it the
" ſixth part of an Inch.

" G G G, the fore Part of the Tube faſtened
" to a Braſs Ring H I, to keep it immoveable.

" P Q K L, the hind Part of the Tube,
" faſtened to another Braſs Ring P Q.

" O, an Iron Hook faſtened to the Ring
" P Q, and furniſhed with a Skrew N,
" thereby to advance or draw back the
" hind Part of the Tube, and ſo by that
" means to put the Specula in their due
" Diſtance.

" M Q G I, a crooked Iron ſuſtaining the
" Tube, and faſtened by the Nail R to the
" Ball and Socket S, whereby the Tube
" may be turned every way.

" The Center of the flat Speculum, CD,
" muſt be placed in the ſame Point of the
" Tube's Axe, where falls the Perpendicular to this Axe, drawn to the ſame from
" the Center of the Eye-Glaſs; which
" Point is here marked at T.

And to give the Reader ſome Satisfaction to underſtand in what Degree it repreſents things diſtinct, and free from Colours, and to know the Aperture by which it admits Light; he may compare the Diſtances of the Focus E from the Vertex's of the little Eye-Glaſs of the concave Speculum; that is, E F $\frac{1}{6}$ of an Inch, and E T U $6\frac{1}{3}$ Inches; and the *Ratio* will be found as 1 to 38: Whereby it appears, that the Objects will be magnified about 38 times. To which Proportion is very conſentaneous an Obſervation of the Crown on the Weather-cock about 200 Feet diſtant: For the Scheme X, *Fig.* 3, repreſents it bigger by $2\frac{1}{2}$ times in Diameter, when ſeen through this, than through an ordinary Teleſcope of about two Foot long.* And ſo ſuppoſing this ordinary one to magnify 13 or 14 times, as by the Deſcription it ſhould, this new one by the Experiment muſt magnify near as much as hath been aſſigned.

Thus far as to the Structure of this Teleſcope. Concerning the metalline Matter, fit for theſe reflecting Speculums, the Inventor has alſo conſidered the ſame; as may be ſeen by two of his Letters, written to the Publiſher from *Cambridge*, *Jan.* 18, and 29, 167½, to this Effect; *viz.*

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* *Fig.* 4.

1. That for a fit metalline Subſtance he would give this Caution; That whilſt Men ſeek for a white, hard, and durable metalline Compoſition, they reſolve not upon ſuch an one as is full of ſmall Pores, only diſcoverable by a Microſcope: For though ſuch an one may to appearance take a good Poliſh, yet the Edges of thoſe ſmall Pores will wear away faſter in the poliſhing than the other Parts of the Metal; and ſo, however the Metal ſeem polite, yet it ſhall not reflect ſuch with an accurate Regularity as it ought to do. Thus Tin-Glaſs mixed with ordinary Bell-Metal makes it more white, and apt to reflect a greater Quantity of Light; but withal its Fumes raiſed in the Fuſion, like ſo many aerial Bubbles, fill the Metal full of thoſe microſcopical Pores. But white Arſenic both blanches the Metal, and leaves it ſolid without any ſuch Pores, eſpecially if the Fuſion hath not been too violent. What the ſtellate *Regulus* of *Mars* (which I have ſometimes uſed) or other ſuch like Subſtance will do, deſerves particular Examination.

To this he adds this further Intimation; That Putty, or other ſuch like Powder, with which it is poliſhed, by the ſharp Angles of its Particles fretteth the Metal, if it be not very fine, and fills it full of ſuch ſmall Holes as he ſpeaketh of. Wherefore Care muſt be taken of that before Judgment be given, whether the Metal be throughout the Body of it porous or not.

2. He not having tried, as he ſaith, many Proportions of the Arſenic and Metal, does not affirm which is abſolutely beſt; but thinks there may conveniently be uſed any Quantity of Arſenic equalling in Weight between the ſixth and eighth Part of the Copper, a greater Proportion making the Metal brittle.

The way which he uſed was this: He firſt melted the Copper alone, then put in the Arſenic; which being melted, he ſtirred them a little together, bewaring in the mean time not to draw in Breath near the pernicious Fumes. After this he put in Tin; and again ſo ſoon as that was melted (which was very ſuddenly) he ſtirred them well together, and immediately poured them off.

He ſaith, He knows not, whether by letting them ſtand longer on the Fire after the Tin was melted, a higher Degree of Fuſion would have made the Metal porous;

but he thought that way he proceeded to be ſafeſt.

He adds, That in that Metal which he ſent to *London* there was no Arſenic, but a ſmall Proportion of Silver; as he remembers, one Shilling in three Ounces of Metal. But he thought withal, that the Silver did as much Harm in making the Metal ſoft, as ſo leſs fit to be poliſhed, as Good in rendering it white and luminous.

At another time he mixed Arſenic one Ounce, Copper ſix Ounces, and Tin two Ounces. And this an Acquaintance of his hath, as he intimates, poliſhed better than he did the other.

As to the Objection, That with this kind of Proſpectives Objects are difficultly found, he anſwers, in another Letter of his to the Publiſher of *Jan.* 6, 167½, That that is the Inconvenience of all Tubes that magnify much; and that after a little Uſe the Inconvenience will grow leſs, ſeeing that himſelf could readily enough find any Day-Objects, by knowing which way they were poſited from other Objects that he accidentally ſaw in it; but in the Night to find Stars, he acknowledges it to be more troubleſome; which yet may, in his Opinion, be eaſily remedied by two Sights affixed to the Iron Rod, by which the Tube is ſuſtained; or by an ordinary Proſpective-Glaſs, faſtened to the ſame Frame with the Tube, and directed towards the ſame Object; as DES-CARTES in his *Dioptrics* hath deſcribed, for remedying the ſame Inconvenience of his beſt Teleſcopes.

So far the Inventor's Letters touching this Inſtrument: Of which having communicated the Deſcription to Monſ. CHRISTIAN HUYGENS *de* ZULICHEM, we received from him an Anſwer to this Effect, in his Letter of *Feb.* 13, 1672, *St. n.*

" I ſee, by the Deſcription you have ſent " me of Mr. NEWTON's admirable Tele" ſcope, that he hath well conſidered the " Advantage which a concave Speculum " hath above convex Glaſſes in collecting " the parallel Rays; which certainly, ac" cording to the Calculation I have made " thereof, is very great. Hence it is, that " he can give a far greater Aperture to " that Speculum, than to an Object-Glaſs " of the ſame Diſtance of the *Focus*; and " conſequently, that he can much more " magnify Objects that way, than by an " ordinary Teleſcope. Beſides, by it he " avoids an Inconvenience, which is inſe" parable from convex Object-Glaſſes, which " is the Obliquity of both their Surfaces; " which vitiateth the Refraction of the Rays " that paſs towards the Sides of the Glaſs, " and does more Hurt than Men are aware

" of. Again; By the mere Reflexion of
" the metalline Speculum, there are not ſo
" many Rays loſt as in Glaſſes; which re-
" flect a conſiderable Quantity by each of
" their Surfaces, and beſides intercept many
" of them by the Obſcurity of their Matter.

" Mean time, the main Buſineſs will be,
" to find a Matter for this Speculum that
" will bear ſo good and even a Poliſh as
" Glaſſes, and a Way of giving this Poliſh
" without vitiating the ſpherical Figure.
" Hitherto I have found no Specula that
" had near ſo good a Poliſh as Glaſs; and
" if M. NEWTON hath not already found
" a Way to make it better than ordinarily,
" I apprehend, his Teleſcopes will not ſo
" well diſtinguiſh Objects as thoſe with
" Glaſſes. But 'tis worth while to ſearch
" for a Remedy to this Inconvenience, and
" I deſpair not of finding one. I believe
" that M. NEWTON hath not been without
" conſidering the Advantage which a para-
" bolical Speculum would have above a
" ſpherical one in this Conſtruction; but
" that he deſpairs, as well as I do, of work-
" ing other Surfaces than ſpherical ones
" with due Exactneſs; though elſe it be
" more eaſy to make a parabolical than el-
" liptical or hyperbolical one, by reaſon
" of a certain Property of the parabolic
" Conoid; which is, that all the Sections

" parallel to the Axis make the ſame Para-
" bola.

Thus far M. HUYGENIUS his judicious Letter; to the latter Part of which, concerning the grinding parabolical Conoids, Mr. NEWTON ſaith, in his Letter to the Publiſher of *Feb.* 20, 1671. That though he with him deſpairs of performing that Work by geometrical Rules, yet he doubts not but that the thing may in ſome meaſure be accompliſhed by mechanical Devices.

To all which I cannot but ſubjoin an Extract of a Letter received very lately (*March* 19) from the Inventor of this new Teleſcope, from *Cambridge*; *viz.*

" In my laſt Letter I gave you Occaſion
" to ſuſpect, that the Inſtrument which I
" ſent you is, in ſome reſpect or other, in-
" diſpoſed, or that the Metals are tarniſh-
" ed; and by your Letter of *March* 16,
" I am fully confirmed in that Opinion: For,
" whilſt I had it, it repreſented the Moon
" in ſome Parts of it as diſtinctly as other
" Teleſcopes uſually do which magnify as
" much as that. Yet I very well know,
" that that Inſtrument hath its Imperfec-
" tions, both in the Compoſition of the
" Metal, and in its being badly caſt; as
" you may perceive by a ſcabrous Place
" near the middle of the Metal of it on the
" poliſhed Side, and alſo in the Figure of
" that

“ that Metal near that ſcabrous Place. And in all thoſe Reſpects that Inſtrument is capable of further Improvement.

“ You ſeem to inſinuate, that the Proportion of 38 to 1 holds only for its magnifying Objects at ſmall Diſtances. But if for ſuch Diſtances, ſuppoſe 500 Feet, it magnifies at that rate, by the Rules of Optics it muſt for the greateſt Diſtance imaginable magnify more than $37\frac{3}{4}$ to 1; which is ſo conſiderable a diminiſhing, that it may be even then as 38 to 1.

“ Here is made another Inſtrument like the former, which does very well. Yeſterday I compared it with a ſix Foot Teleſcope, and found it not only to magnify more, but alſo more diſtinctly. And to day I found that I could read in one of the *Philoſophical Tranſactions*, placed in the Sun's Light, at an hundred Feet Diſtance; and that at an hundred and twenty Feet Diſtance I could diſcern ſome of the Words. When I made this Trial, its Aperture (defined next the Eye) was equivalent to more than an Inch and a third part of the Object-Metal. This may be of ſome Uſe to thoſe that ſhall endeavour any thing in Reflexions; for hereby they will in ſome meaſure be enabled to judge of the Goodneſs of their Inſtruments, *&c.*

Mr. NEWTON's *Letter to the Publiſher, of* March 26, 1672, *containing ſome more Suggeſtions about his new* Teleſcope; *and a Table of* Apertures *and* Charges *for the ſeveral Lengths of that Inſtruments.*

“*SIR*,

“Since my laſt Letter I have further compared the two Teleſcopes, and find that of Metal to repreſent as well the Moon as nearer Objects, ſomething diſtincter than the other. But I muſt tell you alſo, that I am not very well aſſured of the Goodneſs of that other, which I borrowed to make the Compariſon; and therefore deſire that the other Experiment ſhould be rather confided in, of reading at the Diſtance of between 100 and 120 Feet, at which I and others could read with it in the *Tranſactions*, as I found by Meaſure: At which time the Aperture was $1\frac{1}{3}$ of an Inch; which I knew by trying, that an Obſtacle of that Breadth was requiſite to intercept all the Light which came from one Point of the Object.

“I ſhould tell you alſo, that the little plane Piece of Metal near the Eye-Glaſs is not truly figured; whereby it happens, that Objects are not ſo diſtinct at the

" Middle as at the Edges. And I hope,
" that by correcting its Figure (in which
" I find more Difficulty than one would ex-
" pect) they will appear all over diſtinct,
" and diſtincter in the Middle than at the
" Edges. And I doubt not but that the
" Performances will then be greater.

" But yet I find, that there is more Light
" loſt by Reflexion of the Metal, which I
" have hitherto uſed, than by Tranſmiſſion
" through Glaſſes: For which Reaſon a
" ſhallower Charge would probably do
" better for obſcure Objects; ſuppoſe ſuch
" an one as would make it magnify 34 or
" 32 times. But for bright Objects at any
" Diſtance, it ſeems capable of magnifying
" 38 or 40 times with ſufficient Diſtinct-
" neſs. And for all Objects the ſame
" Charge, I believe, may, with Advantage,
" be allowed, if the ſteely Matter, imploy-
" ed at *London*, be more ſtrongly reflec-
" tive than this which I have uſed.

" The Performances of one of theſe In-
" ſtruments of any Length being known,
" it will appear by this following Table
" what may be expected from thoſe of other
" Lengths by this Way, if Art can accom-
" pliſh what is promiſed by the Theory.
" In the firſt Column is expreſſed the Length
" of the Teleſcope in Feet; which doubled

“gives the Semidiameter of the Sphere, on which the concave Metal is to be ground.

Lengths.	Apertures.	Charges.
½	100	100
1	168	119
2	283	141
3	383	157
4	476	168
5	562	178
6	645	186
8	800	200
10	946	211
12	1084	221
16	1345	238
20	1591	254
24	1824	263

“The Uſe of this Table will beſt appear by Example: Suppoſe therefore a half Foot Teleſcope may diſtinctly magnify 30 times with an Inch Aperture, and it being required to know what ought to be the analogous Conſtitution and Performance of a four Foot Teleſcope; by the ſecond Column, as 100 to 476, ſo are the Apertures, as alſo the Number of times which they magnify. And conſequently, ſince the half Foot Tube hath an Inch Aperture, and magnifieth

“ fieth 30 times; a four Foot Tube proportionally
“ ſhould have $4\frac{26}{100}$ Inches Aperture,
“ and magnify 143 times. And
“ by the third Column, as 100 to 168, ſo
“ are their Charges. And therefore if the
“ Diameter of the Convexity of the Eye-
“ Glaſs for a half Foot Teleſcope be $\frac{1}{5}$ of
“ an Inch, that for a four Foot ſhould be
“ $\frac{168}{500}$; that is, about $\frac{1}{3}$ of an Inch.

“ In like manner, if a half Foot Teleſcope
“ may diſtinctly magnify 36 times
“ with $1\frac{1}{4}$ of an Inch Aperture, a four
“ Foot Teleſcope ſhould with equal Diſtinctneſs
“ magnify 171 times with 6 Inches
“ Aperture; and one of ſix Foot ſhould
“ magnify 232 times with $8\frac{2}{3}$ Inches Aperture;
“ and ſo of other Lengths. But
“ what the Event will really be, we muſt
“ wait to ſee determined by Experience:
“ Only this I thought fit to inſinuate, that
“ they which intend to make Trials in
“ other Lengths, may more readily know
“ how to deſign their Inſtruments. Thus
“ for a four Foot Tube, ſince the Aperture
“ ſhould be five or 6 Inches, there will be
“ required a Piece of Metal 7 or 8 Inches
“ broad at leaſt; becauſe the Figure will
“ ſcarcely be true to the Edges: And the
“ Thickneſs of the Metal muſt be proportional
“ to the Breadth, leſt it bend in the
“ grinding. The Metals being poliſhed,

" there may be Trials made with ſeveral " Eye-Glaſſes, to find what Charge may " with beſt Advantage be made uſe of.

An Extract of another Letter of the ſame to the Publiſher, dated March 30, 1672. *by way of Anſwer to ſome Objections made by an ingenious* French *Philoſopher to the new Reflecting* Teleſcope.

" *SIR*,

" I doubt not but M. *A.* will allow the " Advantage of Reflexion in the Theory " to be very great, when he ſhall have informed himſelf of the different Refrangibility of the ſeveral Rays of Light. And " for the practic Part, it is in ſome meaſure manifeſt by the Inſtruments already " made, to what Degree of Vivacity and " Brightneſs a metalline Subſtance may be " poliſhed. Nor is it improbable but that " there may be new Ways of Poliſhing " found out for Metal, which will far excel thoſe that are yet in uſe. And when " a Metal is once well poliſhed, it will be " a long time preſerved from tarniſhing, " if Diligence be uſed to keep it dry, and " cloſe ſhut up from Air: For the principal " Cauſe of tarniſhing ſeems to be, the condenſing of Moiſture on its poliſhed Surface; which by an acid Spirit, where-

“ with the Atmoſphere is impregnated,
“ corrodes and ruſts it; or at leaſt, at its
“ exhaling, leaves it covered over with a
“ thin Skin, conſiſting partly of an earthly
“ Sediment of that Moiſture, and partly of
“ the Duſt, which flying to and fro in the
“ Air, had ſettled and adhered on it.

“ When there is not Occaſion to make
“ frequent Uſe of the Inſtrument, there
“ may be other Ways to preſerve the Metal
“ for a long time; as perhaps by immerging
“ it in Spirit of Wine, or ſome other
“ convenient Liquor. And if they chance
“ to tarniſh, yet their Poliſh may be recovered
“ by rubbing them with a ſoft Piece
“ of Leather, or other tender Subſtance,
“ without the Aſſiſtance of any fretting
“ Powders, unleſs they happen to be ruſty;
“ for then they muſt be new poliſhed.

“ I am very ſenſible, that Metal reflects
“ leſs Light than Glaſs tranſmits; and for
“ that Inconvenience I gave you a Remedy
“ in my laſt Letter, by aſſigning a ſhallower
“ Charge, in proportion to the Aperture,
“ than is uſed in other Teleſcopes. But, as
“ I have found ſome metalline Subſtances to
“ be more ſtrongly reflective, and to poliſh
“ better, and be freer from tarniſhing,
“ than others; ſo I hope there may in time
“ be found out ſome Subſtance much freer
“ from theſe Inconveniences than any yet
“ known.

After Sir ISAAC NEWTON's Teleſcope had been ſent up to the Royal Society, they gave Mr. *Cox* (the Optic-Glaſs-Grinder) Orders to make one after the ſame manner of Contrivance four Foot long; which was done, one End of the Tube being open, and at the other End was placed a concave metalline Mirrour; the Diameter of which was betwixt 4 and 5 Inches; and it was ground on a Sphere of 14 Foot Diameter; and about its Focus, which was about 4 Foot off, was placed a reflecting Plate as big as a Two-pence, inclined at an Angle of 45 Degrees to the Axis: So that the reflected Rays falling thereon were again reflected upright to the Side of the Teleſcope; where the Eye, through a ſmall Hole wherein is placed a ſmall plano-convex Glaſs, beholds the Object on the reflecting Plate as much magnified as it could have been done by an ordinary Teleſcope of 40 Foot long or more, and void of Colours. The Mirrour and reflecting Plate were made to be taken out and wiped at pleaſure. But the Society and *Cox* himſelf were not pleaſed with the Metal or Poliſh of the reflecting Plate; and therefore a Trial was to be made with the *Lapis Oſmandinus*, a black Stone that comes from Mount *Hecla* in *Iſland*, and other Materials, &c. (*Whether or no theſe Trials ſucceeded, I have not been able to* *to learn; but I believe they did not, nor any other, till Mr.* HADLEY *made his* Newtonian *Teleſcope in* 1723.) This Account I copied from a Paper of Mr. JOHN COLLINS; in which was the Copy of Sir ISAAC NEWTON's Letter to a Friend about his Teleſcope (*Orig. Pap.* N°. 2.)

A little after Sir ISAAC NEWTON had ſent his new Teleſcope to the Royal Society, Mr. OLDENBURGH, the Secretary, wrote him a Letter of Thanks; to which Sir ISAAC made Anſwer in 167½, giving a farther Account of the Inſtrument. (*Orig. Pap.* N°. 5.)

About this time Dr. J. GREGORY having an Account of Sir ISAAC NEWTON's Teleſcope, wrote his Thoughts about it to Mr. JOHN COLLINS, in a Letter from *Aberdeen*, dated *Aug. 6*, 1672; in which he gives the Preference to Sir ISAAC's Teleſcope above that which he deſcribed in his *Optica promota* in one reſpect, but thinks his own better in another. (*Orig. Pap.* N°. 3.) At the ſame time one Monſ. CASSEGRAIN publiſhed a Deſcription of a Catadioptric Teleſcope, as his own Invention; which he pretended to have been prior to Sir ISAAC's *Teleſcope*, and which M. BERCE, Publiſher of the *French Memoirs* for the Year 1672, ſeems to prefer to it: But S. SALVETTI, one of the Great Duke's Muſicians at *Florence*, who made one after the manner of Sir Isaac, thinks it much better contrived than M. Cassegrain's. (*Phil. Tranſ.* N°. 87.) Now M. Cassegrain's is not pretended to have been contrived before the Beginning of the Year 1672, and Sir Isaac's was contrived in the Year 1666, and executed in the Year 1670, or at fartheſt finiſhed in the Year 1671: Beſides, M. Cassegrain's differs in nothing from Dr. Gregory's, but that he would have the ſmall Metal to be convex, which Dr. Gregory makes concave; and therefore the Inſtrument ſeems only to be Dr. Gregory's diſguiſed. The whole Account of this, and Sir Isaac Newton's Thoughts upon it, I here ſubjoin, as copied from the *Philoſ. Tranſactions*, N°. 83.

Mr. Isaac Newton's *Conſiderations on Part of a Letter of Monſieur* de Berce, *printed in the eighth* French Memoire, *concerning the* Catadioptrical Teleſcope *pretended to be improved and refined by M.* Cassegrain.

That the Reader may be enabled the better to judge of the whole, by comparing together the Contrivances both of Mr. Newton and M. Cassegrain, it will be neceſſary to borrow from the ſaid *French Memoire* what is there ſaid concerning them; which is as follows.

“ I ſend

“ I ſend you (*ſaith M.* de BERCE *to the Publiſher of the* Memoire) the Copy of the “ Letter which M. CASSEGRAIN hath “ written to me, concerning the Propor- “ tions of Sir SAMUEL MORELAND’s Trum- “ pet. And as for the Teleſcope of Mr. “ NEWTON, it hath as much ſurprized me “ as the ſame Perſon that hath found out “ the Proportions of the Trumpet: For it “ is now about three Months that that Per- “ ſon communicated to me the Figure of a “ Teleſcope, which was almoſt like it, and “ which he had invented; but which I “ look upon as *more witty.** I ſhall here “ give you the Deſcription of it in ſhort. “ ABCD (*Plate* IV. *Fig.* 5.) is a ſtrong “ Tube, in the Bottom of which there is a “ great concave *Speculum* CD, pierced in “ the Middle E.

“ F is a convex *Speculum*, ſo diſpoſed, “ as to its Convexity, that it reflects the “ Species which it receives from the great “ *Speculum* towards the Hole E, where is “ an Eye-Glaſs which one looketh through.

“ The Advantages which I find in this “ Inſtrument above that of Mr. NEWTON, “ are, 1. That the Mouth or Aperture AB “ of the Tube may be of what Bigneſs you “ pleaſe; and conſequently, you may have

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* *More ingenious*, is the Senſe of the *French*.

" many more Rays upon the concave *Speculum*, than upon that of which you have given us the Deſcription. 2. The Reflexion of the Rays will be very natural; ſince it will be made upon the Axis it ſelf, and therefore more vivid. 3. The Viſion of it will be ſo much the more pleaſing; in that you ſhall not be incommoded by the great Light, by reaſon of the Bottom C D, which hideth the whole Face. Beſides that you'll have leſs Difficulty in diſcovering the Objects, than in that of Mr. NEWTON.

So far this *French* Author. To which we ſhall now ſubjoin the Conſiderations of Mr. NEWTON, as we received them from him in a Letter written from *Cambridge*; *May* 4, 1672. as follows.

SIR,

" I ſhould be very glad to meet with any Improvement of the Catadioptrical Teleſcope; but that Deſign of it, which (as you inform me) Mr. CASSEGRAIN hath communicated three Months ſince, and is now printed in one of the *French Memoirs*, I fear will not anſwer Expectation: For, when I firſt applied myſelf to try the Effects of Reflexion, Mr. GREGORY's *Optica promota* (printed

“ in the Year 1663) being fallen in my
“ Hands, where there is an Inſtrument
“ (deſcribed Page 94) like that of Monſ.
“ CASSEGRAIN's, with a Hole in the midſt
“ of the Object-Metal to tranſmit the Light
“ to an Eye-Glaſs placed behind it; I had
“ thence an Occaſion of conſidering that
“ Sort of Conſtructions, and found their
“ Diſadvantages ſo great, that I ſaw it ne-
“ ceſſary, before I attempted any thing in
“ the Practice, to alter the Deſign of them,
“ and place the Eye-Glaſs at the Side of
“ the Tube, rather than at the Middle.

“ The Diſadvantages of it you will un-
“ derſtand by theſe Particulars: 1. There
“ will be more Light loſt in the Metal by
“ Reflexion from the little convex *Specu-*
“ *lum*, than from the oval Plane: For it
“ is an obvious Obſervation, that Light is
“ moſt copiouſly reflected from any Sub-
“ ſtance when incident moſt obliquely.
“ 2. The convex *Speculum* will not reflect
“ the Rays ſo truly as the oval Plane, un-
“ leſs it be of an hyperbolic Figure; which
“ is incomparably more difficult to form
“ than a plane; and if truly formed, yet
“ would only reflect thoſe Rays truly which
“ reſpect the *Axis*. 3. The Errors of the
“ ſaid Convex will be much augmented by
“ the too great Diſtance through which the
“ Rays reflected from it muſt paſs, before
“ their

“ their Arrival at the Eye-Glaſs. For which Reaſon I find it convenient to make the Tube no wider than is neceſſary, that the Eye-Glaſs be placed as near to the oval Plane as is poſſible, without obſtructing any uſeful Light in its Paſſage to the Object-Metal. 4. The Errors of the Object-Metal will be more augmented by Reflexion from the Convex than from the Plane, becauſe of the Inclination or Deflexion of the Convex on all Sides from the Points on which every Ray ought to be incident. 5. For theſe Reaſons there is requiſite an extraordinary Exactneſs in the Figure of the little Convex; whereeas I find by Experience, that it is much more difficult to communicate an exact Figure to ſuch ſmall Pieces of Metal, than to thoſe that are greater. 6. Becauſe the Errors at the Perimeter of the concave Object-Metal, cauſed by the Sphericalneſs of its Figure, are much augmented by the Convex, it will not with Diſtinctneſs bear ſo large an Aperture as in the other Conſtructions. 7. By reaſon that the little Convex conduces very much to the magnifying Virtue of the Inſtrument, which the oval Plane doth not, it will magnify much more in proportion to the Sphere on which the great Concave is ground, than in the

“ other Deſign; and ſo magnifying Objects much more than it ought to do in proportion to its Aperture, it muſt repreſent them very obſcure and dark; and not only ſo, but alſo confuſed, by reaſon of its being overcharged. Nor is there any convenient Remedy for this: For if the little Convex be made of a larger Sphere, that will cauſe a greater Inconvenience, by intercepting too many of the beſt Rays; or, if the Charge of the Eye-Glaſs be made ſo much ſhallower, as is neceſſary, the Angle of Viſion will thereby become ſo little, that it will be very difficult and troubleſome to find an Object; and of that Object, when found, there will be but a very ſmall Part ſeen at once.

“ By this you may perceive, that the three Advantages which Monſ. CASSEGRAIN propounds to himſelf, are rather Diſadvantages: For, according to his Deſign, the Aperture of the Inſtrument will be but ſmall, the Object dark and confuſed, and alſo difficult to be found. Nor do I ſee why the Reflexion is more upon the ſame *Axis*, and ſo more natural in the one Caſe than in the other; ſince the *Axis* itſelf is reflected towards the Eye by the oval Plane; and the Eye may be defended from external Light as "well at the Side as at the Bottom of the "Tube.

"You ſee therefore that the Advantages "of this Deſign are none, but the Diſad- "vantages ſo great and unavoidable, that "I fear it will never be put in Practice with "good Effect. And when I conſider, that "by reaſon of its Reſemblance with other "Teleſcopes, it is ſomething more obvious "than the other Conſtruction, I am apt "to believe that thoſe who have attempted "any thing in Catoptrics, have ever tried "it in the firſt place; and that their bad "Succeſs in that Attempt hath been the "Cauſe why nothing hath been done in "Reflexions: For Mr. GREGORY ſpeaking "of theſe Inſtruments in the aforeſaid Book, "Page 95. ſaith, *De Mechanica horum "Speculorum & Lentium ab aliis fruſtra "tentatâ, ego in Mechanicis minus verſa- "tus nihil dico.* So that there have been "Trials made of theſe Teleſcopes, but yet "in vain. And I am informed that about "ſeven or eight Years ſince, Mr. GREGORY "himſelf, at *London*, cauſed one of ſix "Foot to be made by Mr. *Reive*; which I "take to have been according to the afore- "ſaid Deſign deſcribed in his Book, be- "cauſe though made by a skilful Artiſt, yet "it was without Succeſs.

"I could wiſh therefore Mr. CASSEGRAIN "had tried his Deſign before he divulged "it: But if, for further Satisfaction, he "pleaſe hereafter to try it, I believe the "Succeſs will inform him, that ſuch Pro-"jects are of little Moment till they be put "in Practice.

After this Dr. GREGORY, in a Letter to Mr. COLLINS from St. *Andrews*, *Sept*. 23. 1672 (N°. 4.) replies to Sir ISAAC's Animadverſions upon M. CASSEGRAIN's Teleſcope, looking upon it as his own diſguiſed; and propoſes to uſe a plane *Speculum* inſtead of his concave or CASSEGRAIN's convex, to remedy ſome Inconveniences found by Sir ISAAC, and make his Teleſcope ſtill ſhorter than Sir ISAAC's. He thinks the oblique Reflexion from the little Plate in Sir ISAAC's much worſe than his own direct Reflexion from his ſmall *Speculum*; but thinks a little Concave or Convex worth trying, becauſe different Charges may be then given to the Teleſcope with the ſame Eye-Glaſs; which he thinks impracticable in Sir ISAAC's. Sir ISAAC's Anſwer to theſe Objections was ſent to Mr. COLLINS, which he communicated to Mr. GREGORY (*but I could not meet with that Letter of Sir* ISAAC's) who ſeems thereby convinced, that an oblique Reflexion is preferable to a direct one; but does not conceive how Sir ISAAC can alter the Charge of his Teleſcope without changing the Eye-Glaſs.

(N. B. *Dr.* GREGORY's *Letter which contains this Reply is printed at the End of the* Appendix, No. 6. *But I don't take notice here of the Arguments* pro *and* con *concerning the* Tubercula, *or little Eminences in the reflecting ſmall Metal being ſtruck directly or obliquely by the Rays of Light; becauſe in fact they are not ſtruck at all by the Rays of Light, which are reflected without Contact or Impulſion againſt the* Tubercula, *or any Part of the poliſhed Metal; as Sir* ISAAC *found out afterwards, though he did not know it then.*)

Sir ISAAC in his next Letter (the laſt we publiſh here of his, N°. 7.) gives an Account of his Manner of varying the Charge in his Teleſcope, by making uſe of a Glaſs Priſm with two convex Surfaces and a flat one, inſtead of his ſmall plane Mirrour. He alſo ſhews how the Aperture of the Eye-Glaſs ought to be limited in Dr. GREGORY's Teleſcope; and for his own Teleſcope he lays all the Streſs of Magnifying upon the Eye-Glaſs. The reſt of the Letter is concerning a Reflecting Microſcope mentioned by him in the *Tranſactions*, and a reflecting Glaſs Mirrour for burning, propoſed by Dr. GREGORY in one of his Letters (N°. 6.) which Mirrours are now very commonly made.

Dr. GREGORY in his laſt Letter to Mr. COLLINS on this Subject, from St. *Andrews*, *May* 13, 1673. (N°. 8.) commends Sir ISAAC NEWTON's Way of varying the Charge of his Teleſcope; but thinks it liable to ſome Errors, owing to the Refraction of the Rays at their Entrance into and Emerſion out of the convex Surfaces of the little Priſm. He thinks it not worth while to look on terreſtrial Bodies with excellent Teleſcopes, becauſe they magnify the Particles of the Atmoſphere as well as the Object. Then he propoſes a Plate of Metal with a ſmall Hole in it to be placed in the *Focus* of his Eye-Glaſs next to the Eye, to intercept all the ſpurious Rays (*without which, as it has appeared ſince in Practice, this Teleſcope would be of no Uſe*) and concludes with aſſerting, that if it appears by Trials that common Microſcopes (that is Microſcopes with two or three Glaſſes) can be brought to exceed all Improvements of a ſingle *Lens*, then it will follow from thence that his Teleſcope may be brought to exceed Sir ISAAC NEWTON's.

There have been no Attempts made ſince that time (*viz.* ſince the Year 1673.) to make either of theſe Sorts of Teleſcopes,* at leaſt no ſuccesful ones that have been made publick, till in the Year 1723, that is at the Diſtance of fifty Years, Mr. HADLEY made the reflecting Teleſcope deſcribed in the *Philoſophical Tranſactions*, N°. 376, to which I refer the Reader: But I think it not improper to copy from another *Tranſaction* (N°. 378.) the Obſervations made by the late Reverend Mr. POUND, and his Nephew the Reverend Mr. BRADLEY, at *Wanſtead*, in the Year 1723. as alſo others made by Mr. HADLEY himſelf with this Teleſcope, before I give an Account alſo of the *Gregorian* Teleſcope, now likewiſe made and brought to Perfection by the ſaid Mr. HADLEY, together with a Table of his Calculations concerning this Teleſcope, communicated to me lately.

Phil. Tranſ. N°. 378. *Pag.* 382.

A Letter from the Rev. Mr. JAMES POUND, *Rector of* Wanſtead, *F. R. S. to Dr.* JURIN, *Secr. R. S. concerning Obſervations made with Mr.* HADLEY'*s* Reflecting Teleſcope.

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* Three or four *French* Authors have indeed mentioned the Uſe of concave metalline Mirrours, inſtead of one of the Eye-Glaſſes of a Dioptrical Teleſcope; but the Thing was never put in Practice.

“ It were to be wiſhed, that, with the particular Deſcription given in a late *Tranſaction* (N°. 376.) of the curious Mechaniſm of that Catadioptric Teleſcope which was made by Mr. HADLEY, and by him preſented to the *Royal Society*, that moſt ingenious Gentleman would have communicated alſo a full Account of what Obſervations he had made with it; whereby the Publick might at length have been apprized of the Uſefulneſs of an Invention (worthy of its great Author Sir ISAAC NEWTON) which, perhaps from the vain Attempts made by ſome of putting it in Practice, hath lain neglected theſe fifty Years: For it is ſo long ſince it was firſt publiſhed in the *Philoſophical Tranſactions*, N°. 81.

“ Mr. HADLEY hath ſufficiently convinced us, that this noble Invention doth not conſiſt in bare Theory; and it is to be hoped, that he, or ſome other ſuch curious and worthy Perſons (who ſcruple not at a little Pains and Coſt) will in a ſhort time find out a Method, either of preſerving the concave Metal from tarniſhing, or of clearing it eaſily when tarniſhed, or elſe of making a good concave *Speculum* of Glaſs quickſilvered on the back part. When a Method for either of theſe ſhall be diſcovered, 'tis not to be

“ doubted, but that the old Dioptric Teleſcope will be for the moſt part laid by, and this Catoptric one will be chiefly in uſe among the practical Aſtronomers; inaſmuch as ſeveral Inconveniences and Difficulties, which are unavoidable in the Management of the former, eſpecially when long, are in this latter wholly avoided.

“ It is no ſmall Convenience, that, by means of one of theſe Reflecting Teleſcopes, whoſe Length exceeds not five Feet (and which may be managed at a Window within the Houſe) celeſtial Objects appear as much magnified, and as diſtinct, as they do through the common Teleſcope of more than a hundred Feet in Length.

“ Mr. Bradley, the *Savilian* Profeſſor of Aſtronomy, and myſelf, have compared Mr. Hadley's Teleſcope (in which the focal Length of the Object-Metal is not quite 5 Feet and $\frac{1}{4}$) with the *Huygenian* Teleſcope, the focal Length of whoſe Object-Glaſs is 123 Feet: And we find, that the former will bear ſuch a Charge, as to make it magnify the Object as many times as the latter with its due Charge; and that it repreſents Objects as diſtinct, tho' not altogether ſo clear and bright; which may be occaſioned partly from the

" Difference of their Apertures (that of the " *Huygenian* being ſomewhat the larger) " and partly from ſeveral little Spots in the " concave Surface of the Object-Metal, " which did not admit of a good Poliſh.

" Notwithſtanding this Difference in the " Brightneſs of the Objects, we were able, " with this Reflecting Teleſcope, to ſee " whatever we have hitherto diſcovered " by the *Huygenian*; particularly the Tranſits of *Jupiter*'s Satellites, and their " Shades, over the Disk of *Jupiter*; the " black Liſt in *Saturn*'s Ring, and the " Edge of the Shade of *Saturn* caſt on his " Ring, as repreſented by *Fig.* 4. *Plate* " II. of the forementioned *Tranſaction*, " N°. 376.

" We have alſo ſeen with it ſeveral " times the five Satellites of *Saturn*; in " viewing of which this Teleſcope had the " Advantage of the *Huygenian*, at that " time when we compared them: For it being in Summer, and the *Huygenian* Teleſcope being managed without a Tube, " the Twilight prevented us from ſeeing in " this ſome of thoſe ſmall Objects, which " at the ſame time we could diſcern with " the reflecting Teleſcope.

" *I am*, &c.

" Ja. Pound.

Obſerva-

Obſervations on the Satellites of Jupiter *and* Saturn, *made with the ſame* Teleſcope. *By* JOHN HADLEY, *Eſq; F. R. S. Extracted from the Minutes of the* Royal Society, April 6. 1721.

" Mr. HADLEY gave the *Society* a Re-
" lation of ſome of the moſt remarkable
" Obſervations, which he had made with
" his Reflecting Teleſcope, before he pre-
" ſented it to the *Society*.

" In obſerving *Jupiter*'s Satellites, he
" has ſeen diſtinctly the Shadows of the
" firſt and third Satellites caſt upon the
" Body of the Planet: Mr. FOLKES and
" Dr. JURIN, being preſent, affirmed, that
" Mr. HADLEY had likewiſe ſhewn them
" the Shadow of the third Satellite through
" the ſame Teleſcope.

" In obſerving *Saturn* the laſt Spring,
" at a Time when that Planet was about 15
" Days paſt the Oppoſition, he ſaw the
" Shade of the Planet caſt upon the Ring,
" and plainly diſcerned the Ring to be diſ-
" tinguiſhed into two Parts, by a dark
" Line, concentric to the Circumference of
" the Ring: The outer or upper Part of
" the Ring ſeemed to be narrower than the
" lower or inner Part, next the Body; and
" the dark Line, which ſeparated them,

" was ſtronger next the Body, and fainter
" on the outer Part towards the upper
" Edge of the Ring. Within the Ring he
" diſcerned two Belts; one of which croſſed
" *Saturn* cloſe to its inner Edge, and ſeemed
" like the Shade of the Ring upon the
" Body of *Saturn*: But when he conſidered
" the Situation of the Sun, in reſpect to
" the Ring and *Saturn*, he found that Belt
" could not ariſe from ſuch a Cauſe.

" He ſays, that at Times he has ſeen with
" this Teleſcope three different Satellites
" of *Saturn*; but could never have the
" Fortune to ſee all five.

" *Aug.* 1723. Mr. HADLEY adds, that
" he has ſeveral times ſeen the Shadow of
" the firſt, ſecond, and third Satellites of
" *Jupiter* paſs over the Body of that Planet;
" and that he has ſeen the firſt and
" ſecond appear, as a bright Spot upon the
" Body of *Jupiter*, and has been able to
" keep Sight of them there for about a
" Quarter of an Hour, from the Time of
" their entering on his Limb.

" *Jupiter*'s Satellites have of late Years
" been ſo ſituated, with regard to the
" Earth and *Jupiter*, that he has not had
" ſufficient Opportunity of obſerving the
" Tranſit of the fourth Satellite, or of its
" Shadow.

“ The dark Line on the Ring of *Saturn*, “ parallel to its Circumference, is chiefly “ viſible on the *Anſæ*, or Extremities of the “ elliptic Figure, in which the Ring ap- “ pears; but he has ſeveral times been able “ to trace it very near, if not quite round; “ particularly in *May* 1722. he could diſ- “ cern it without the Northern Limb of “ *Saturn*, in that Part of the Ring that “ appeared beyond the Globe of the Planet. “ The Globe of *Saturn* (at leaſt towards “ its Limb) reflects leſs Light than the in- “ ner Part of the Ring; and he has ſome- “ times diſtinguiſhed it from the Ring by “ the Difference of Colour.

“ The duſky Line, which in 1720 he “ obſerved to accompany the inner Edge of “ the Ring croſs the Diſk, continues cloſe “ to the ſame, though the Breadth of the “ Ellipſe is conſiderably increaſed ſince “ that time.

An Account of the *Gregorian Reflecting Teleſcope*, as perfected, by JOHN HADLEY, Eſq; *Vice-Preſident* of the *Royal Society*, in the Year 1726.

At the Bottom of the large Tube (expreſſed by a ſtrong black Line) is fixed the concave metalline *Speculum* BB, perforated with the Hole CC; oppoſite to which

Hole is placed a ſmall *Speculum* of the ſame Metal F F cconcave towards the great Metal, and ſo fixed to a crooked Arm, that it may be brought towards or carried fromwards the great *Speculum*, keeping its *Axis* ſtill in the ſame Line (*viz.* in the common *Axis* of both *Specula*) and by that means parallel Rays, or Rays from the Points of a very diſtant Object, coming to the great *Speculum* in the Lines O O, P P, *&c.* and falling upon the great *Speculum* between B and C will be ſo united at its *Focus*, as to form there the Image G G of the Object O P ſuppoſed at a vaſt Diſtance. The Rays diverging again from their reſpective Points of the Image, go on diverging, and fall upon the little Concave F F, whoſe focal Length is D I, and from its Surface are reflected nearly parallel to their reſpective *Axes* (not wholly ſo, becauſe D G is greater than D I) and with all the *Axes* or principal Rays move parallel to the common *Axis* through the Hole in the great *Speculum*, in the Direction D A, ſo entering into the ſmall Tube N M M N, which is fixed to the great Tube behind the *Speculum*, fall upon the convex Side of the plano-convex ocular *Lens* N N, and paſſing through it, form a ſecond Image at *g g*, whoſe Bigneſs is limited by the Hole of the perforated opake Circle or Diaphragm placed at RR. That ſecond or erect Image of the firſt inverted Image of an erect Object is ſeen large, clear, and diſtinct by the Eye at E, which ſees it through the ſmall Hole in the Plate MM, and the laſt Eye-Glaſs SS, which is a *Meniſcus*: For the Eye will ſee it under the Angle SES made by the *Axes* of thoſe Pencils of Rays which came from the Extremities of the viſible Object; and the Rays belonging to each Pencil will be parallel to their reſpective *Axes*. Beſides, the ſpurious Rays will be all cut off by the Plate MM; which makes the Viſion very diſtinct. This Teleſcope is not only good for common Eyes, but the Rays that enter the Eye will be made to converge a little for the *Presbytæ*, or to diverge a little for the *Myopes*, by means of a Skrew fixed to the Arm of the little Concave, to remove or to bring it forward upon Occaſion. [N. B. *The Reaſon why Mr.* HADLEY *uſes a double Eye-Glaſs inſtead of a ſingle one propoſed by Dr.* GREGORY, *is, to prevent the object being coloured at the Edges of the Aperture.*] The ſeventh Figure is drawn in the quarter Proportion of an Inſtrument of 12 Inches focal Length.

BB is the larger concave *Speculum*, its focal Length AG.

F F is the ſmaller concave *Speculum*, its focal Length I D.

The Breadth F F is about $\frac{1}{50}$ of an Inch wider than the Hole C C in the larger *Speculum.*

N the firſt Eye-Glaſs is plano-convex.

S the ſecond Eye-Glaſs plano-convex likewiſe, or rather a *Meniſcus.*

M a Plate with a ſmall Hole in it to exclude all foreign Light, with an Hole in it $\frac{1}{18}$ of an Inch.

R R the limiting Circle or Diaphragm.

The Arrows are the ſeveral ſucceſſive Images of any Object.

TABLE I.

If A G be	= 3 Inches	= 12 Inches	= 18 Inches	= 27 Inches	
And the Charge is	$12\frac{2}{3}$ or 13	= 36	= 49	= 66	That is, the Power of Magnifying.
B B	= 0. 7	= 2.	= 2. 7	= 3. 7	
I D	= 0. 82	= 2.32	= 3.22	= 4.22	
F F	= 0.315	= 0.56	= 0. 7	= 0.88	
C C	= 0.295	= 0.54	= 0.68	= 0.86	
Focal Length of N	= 1. 48	= 3.27	= 3.97	= 4.91	
Focal Length of S	= 0. 7	= 1. 3	= 1.54	= 1.85	
A D	= 3. 96	= 14.66	= 21.69	= 31. 9	
A N	= 0. 5	= 0. 7	= 0.75	= 0. 8	
N S	= 1. 4	= 2. 6	= 3.08	= 3. 7	
S M	= 0. 45	= 0.76	= 0.88	= 1. 0	
R R	= 0. 2	= 0.37	= 0.44	= 0.53	

N. B. Theſe Calculations are for the Day, where Objects are to be magnified but little in proportion to what the Heavenly Bodies may be at Night; for which the following Table gives the Proportions.

TABLE II.

If A G be = 12 Inches = 18 Inches = 27 Inches;
And the Charge 70 = 95 = 128

B B muſt be = 2. = 2. 7 = 3.75
I D = 1.74. = 2.36 = 3,22
F F = 0. 4. = 0.47 = 0.56
C C = 0.38. = 0.45 = 0.54
Focus of N = 2.29. = 2.79 = 3.47
Focus of S = 0.87. = 1.03 = 1.25
A D = 13.95. = 20.64 = 30.56
A N = 0. 7. = 0.75 = 0. 8
N S = 1.74. = 2.06 = 2. 5
S M = 0.47. = 0.56 = 0. 7
R R = 0.25. = 0.29 = 0.36

The Breadth in the Hole of the Plate M muſt be $\frac{1}{35}$ of an Inch.

N. B. The varying the Length of A N, that is the Diſtance of the firſt Eye-Glaſs behind the fore Surface of the *Speculum* B B, alters the other Proportions but little; ſo that if the Thickneſs of the *Speculum*, or other Circumſtances require it, there is no need to keep exactly to the Numbers here ſet down for it.

Theſe Teleſcopes are much more convenient for Day-Objects, and more eaſily managed by Perſons who are not uſed to Teleſcopes in general, than the *Newtonian* Teleſcope: But again, they are not ſo convenient for Celeſtial Objects, eſpecially ſuch as are at at a conſiderahle Altitude in the Heavens; for Celeſtial *Phænomena* may be viewed quite up to the Zenith with the ſame Conveniency as at the Horizon, with Sir ISAAC's Teleſcope; which cannot be done with the *Gregorian*. Beſides, in looking at Celeſtial Objects, we may ſee them only with an Eye-Glaſs, and by means of one ſingle Image, in uſing the *Newtonian* Teleſcope; whereas the *Gregorian* has always two Images.

In ſhort, when once Micrometers can be uſefully applied to theſe two Sorts of Reflecting Teleſcopes (which Artiſts are now endeavouring to do) the long Dioptrical Teleſcopes will be quite out of uſe, and Aſtronomical Obſervations will be made as certainly, much cheaper, and with more Expedition.

N. B. The *two* foregoing *Tables* of *Calculation*, concerning the *Gregorian Teleſcope*, being only ſent me, by Mr. HADLEY, for my private Aſſiſtance in making that Inſtrument; upon my asking Leave to publiſh it, he anſwered, That as he did not deſign to make it public, he had not been correct, farther than the *ſecond* or *third* Place of the *Decimals*; and therefore could not give it out for perfect. But as it is ſufficient for Practice, I thought proper to give it here. And, at the Cloſe of this *Appendix*, I have added Mr. HADLEY's Letter to me with his new Calculations for the *Reflecting Teleſcope*.

I ſhould do Injuſtice to Mr. HADLEY, if I ended this *Appendix* without taking notice of his admirable new Invention of a *Reflecting Quadrant*, for taking Altitudes and Angular Diſtances more accurately than ever yet was done; and with ſo much Eaſe, that the Shake of a Ship does not influence the Correctneſs of the Obſervation, it being only required to find the Object. This is an Advantage which no Quadrant ever yet had. I do not here give the Deſcription of it, becauſe it is fully done in the *Philoſophical Tranſactions* (N°. 420.) and the *Quadrant* may be had at Mr. B. SISSON's, the Corner of *Beaufort-Buildings* in the *Strand*; where a Book is given with the *Quadrant*, which ſhews its Uſe in the plaineſt manner. As ſoon as the common Prejudice againſt new Things is worn off, and the Inſtrument is well known, I do not believe any Ship will go a long Voyage without one of theſe excellent *Quadrants*.

ORIGINALS Referred to in the foregoing APPENDIX.

NUMB. I.

Ex J. GREGORII Optica promota, *P.* 93.

—— *Tertius autem genus aureum nulla habet incommoda, & omnes priorum generum proprietates habere poteſt*, &c.

THE third Sort, which from its Value we may very well call a *golden one*, has no Inconveniences, and may have all the Properties of the other Sorts; provided the *Lens's* and *Specula* be rightly diſpoſed; that is, if the laſt Image and the laſt but one be produced by *Specula*, and the reſt by *Lens's*. We ſhall therefore here, for Example, deſcribe a Teleſcope of this moſt perfect Kind. Let ADCE (*Pl.* IV. *Fig.* 1.) be a parabolical concave *Speculum* moſt exquiſitely poliſhed; *in whoſe Focus* * C is placed a ſmall elliptic concave *Speculum*, having a common *Focus* and common *Axis* with the concave parabolic *Speculum*, and let it be fixed in that Situation. Now the ſaid *Focus* of that elliptic *Speculum* muſt be very near to its *Vertex*, and the other *Focus* of it muſt be very far from the ſame at F, in the common *Axis* produced beyond the parabolic *Speculum*; and thro' the *Vertex* of the parabolic *Speculum* muſt be made a round Hole M N, in which Hole muſt be placed a Tube having the ſame *Axis* with the *Specula*, and big enough to receive the Rays of a Viſible reflected from the elliptic concave *Speculum*; and let it be continued to L very near to F; and at L let a *Lens* of Cryſtal, convex towards the *Specula*, with the Convexity of a Conoid, and the Denſity of the Cryſtalline (of the Eye) be fixed, whoſe exterior *Focus* muſt be at F, and which muſt be plane towards the Eye, and likewiſe have the ſame common *Axis* as the *Specula* and the Tube. This will be the Way to make an excellent Teleſcope for *Presbytæ*: For diſtant Objects ſeen through the Tube will appear very diſtinctly, magnified very near in the *Ratio* of the Diſtances of the *Vertices* from the common *Foci*; and enlightened in the ſame manner as a viſible would be, when ſeen under ſuch an Angle; provided the Diameter of what produces the laſt Image be big enough to ſuffer the *Uvea* of the Eye to be filled with the Rays: And how that may be done, we have taught in the *Scholium* of the 51ſt of this Book, *&c.*

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* The Author means, *Near whoſe Focus.*

NUMB. II.

Copy of a Letter written by Mr. NEWTON *to a Friend of his* (taken from Mr. *Collins*'s Tranſcript.)

Trin. Coll. Cambridge,
Feb. 23, 166$\frac{1}{9}$.

I Promiſed in a Letter to Mr. *Ent* to give you an Account of my Succeſs in a ſmall Attempt I had then in hand; and it is this: Being perſuaded of a certain Way whereby the practical Part of Optics might be promoted, I thought it beſt to proceed by degrees, and make a ſmall Proſpective firſt, to try whether my Conjecture would hold good or not. The Inſtrument that I made is but ſix Inches in Length; it bears ſomething more than an Inch Aperture, and a plano-convex Eye-Glaſs, whoſe Depth is $\frac{1}{6}$ or $\frac{1}{7}$ Part of an Inch: So that it magnifies about 40 times in Diameter; which is more than any ſix Foot Tube can do, I believe, with Diſtinctneſs. But by reaſon of bad Materials, and for want of good Poliſh, it repreſents not Things ſo diſtinct as a ſix Foot Tube will do; yet I think it will diſcover as much as any three or four Foot Tube, eſpecially if the Objects be luminous. I have ſeen with it *Jupiter* diſtinctly round and his Satellites, and *Venus* horned. Thus, Sir, I have given you a ſhort Account of this ſmall Inſtrument; which though in itſelf contemptible, may yet be looked upon as an Epitome of what may be done according to this Way: For I doubt not but in time a ſix Foot Tube may be made after this Method, which will perform as much as any 60 or 100 Foot Tube made after the common Way; whereas I am perſuaded, that, were a Tube made after the common Way, of the pureſt Glaſs exquiſitely poliſhed with the beſt Figure that any Geometrician (*Des-Cartes*, &c.) hath or can deſign; (which I believe is all that Men have hitherto attempted or wiſhed for) yet ſuch a Tube would ſcarce perform as much more as an ordinary good Tube of the ſame Length. And this however it may ſeem a paradoxical Aſſertion, yet it is the neceſſary Conſequence of ſome Experiments which I have made concerning the Nature of Light, *&c.*

NUMB. III.

Copy of Part of a Letter to Mr. JOHN COLLINS.

Aberdeen, 6 *Auguſt*, 1672.

——IT is like indeed that Mr. NEWTON his Teleſcope may have an Advantage above that which I mention'd in my *Optica promota*, becauſe the Eye-Glaſſe is ſo near the plane Mirroir; yet the Obliquitie of the Mirroir hindereth ſomeqt: Nevertheleſs my Teleſcope hath one Advantage alſo verie conſiderable; for the ſame concave Mirroir togither with the ſame plane-convex Eye-Glaſſe may give the ſame Object-Mirroir any deſired Charge, *&c.*

Your humble and
obliged Servant,

J. GREGORIE.

NUMB. IV.

St. Andrews, 26 *Sept.* 1672.

SIR,

YOurs of the third of *Auguſt* I received a conſiderable time ago, *&c.* — I have caſt an Eye on Mr CASSEGRAIN his Teleſcope,

ſcope, which ſeemeth to be the ſame with that in my *Optica promota*, Page 94. onlie he hath a convex Speculum F (*Plate* IV. *Fig.* 5.) in place of my concave; which is no great Alteration. I think myſelf therfor obliged to anſwer to theſe Diſadvantages Mr. NEWTON finds in it. I mak therfor F an plane Speculum, and then almoſt the whol Diſadvantages evaniſhe, except onlie the third; and for that, ther is an Advantage as conſiderable, if not more; *viz.* that the Diſtance E F groweth almoſt the one halfe leſſe; and therfor the Errors of the concave C D ar alſo diminiſhed upon the plane F by one halfe. Ther is yet another Advantage of this Teleſcope, that it will be little more then halſe the Lenth of Mr. NEWTON's, and doe the ſame Effect. Nevertheleſs of theſe Diſadvantages which Mr. NEWTON mentioned, even with a concave or convex Speculum, this Teleſcope may be worth the trying; ſeing the Eye-Glaſſe and Speculum F being moveable, the Speculum C D can have by their Help anie deſirable Charge; which I think a great Advantage. What I ether did or ſaid needeth not diſcourage anie; for I ſpeak ther onlie of the hyperbolick and elliptick Glaſſes and Speculum, which wer attempted in vaine, as it is clear from the Senſe of the Word. As for my Experiment with Mr. *Rives*,

Rives, he could not poliſh the large Concave upon the Tool: And I (not knowing anie Advantage of the Catoptrick Teleſcope above the Dioptrick, ſave onlie the Shortnes and Similitude betwixt the Circle and Parabola, which is greater than that betwixt the Circle and Hyperbola) imagined that this great Defect in the Figure wold eaſilie counterbalance theſe two pettie Advantages. Upon this Account, and being about to go abroad, I thought it not worth the Pains to truble my ſelf anie further with it; ſo that the Tube was never made: Yet I made ſome Tryals both with a litle concave and convex Speculum; which wer but rude, ſeing I had but tranſient Views of the Object; being ſo poſſeſſed with the Fancie of the defective Figure, that I wold not be at the Pains to fix everie thing in its due Diſtance. Ther is no ſuch Exactnes required in the Speculum F as in the Speculum C D; but indeed more than in the Eye-Glaſſe. I ſupoſe ther is no Queſtion that direct Rayes have the Advantage of oblique; ſeing a Ball thrown directlie on a rough Wall hath a more regular Reflection then when it is thrown obliquelie. However this is not derogat from Mr. Newton, whoſe Diſcoveries hath made the Catoptrick Teleſcopes preferable to the Dioptrick, *&c.*

This Teleſcope with the plane Speculum will indeed loſe maire of the beſt Raies; but theſe nevertheleſs ar always leſs then $\frac{1}{4}$ of the whol, the Eye-Glaſſe being advantagiouſlie ſituat; which Defect ſome perchance may think recompenſed by the Shortnes of the Teleſcope.

I ſuppoſe ther is no great Hazard of overcharging the Teleſcope by the concave or convex Speculum; for the Charge can be changed at Pleaſur: Nether is it probable to me that the Errors of the Object-Speculum ar made more ſenſible (the magnification being alwaies the ſame) by a concave or convex Speculum and an Eye-Glaſſe, then by an plane Speculum and an Eye-Glaſſe, ſave onlie upon the Account of greater Diſtances; which I think the onlie Defect of this Teleſcope.

Your humble Servant,

J. GREGORIE.

NUMB. V.

Cambridge, Jan 6, 167$\frac{1}{2}$.

SIR,

AT the reading of your Letter I was ſurpriſed to ſee ſo much Care taken about ſecuring an Invention to me of which I have hitherto had ſo little Value. And therefore ſince the *Royal Society* is pleaſed to think it worth the patronizing, I muſt acknowledg it deſerves much more of them for that; than of me, who, had not the communication of it been deſired, might have let it ſtill remained in private, as it hath already done ſome Yeares.

— The Deſcription of the Inſtrument you ſent me is very well, only the radius of the concave metal, which you put 14 Inches, is more juſtly 12 $\frac{2}{3}$ or 13 Inches; and the radius of the eye-glaſs, which you put half an inch, is the twelft part of it, if not leſs: For the metal collects the ſun's rays at 6$\frac{1}{3}$ inches diſtance, and the eye-glaſs at leſs than $\frac{1}{6}$ part of an Inch Diſtance from its vertex. By the tools alſo to which they were ground I know their Dimenſions; and particularly meaſuring the Diameter of the hemiſphærical Concave, in which the Eye-Glaſs was ground, I find it the ſixth part of an Inch.

Perhaps it may give ſome Satisfaction to Mounſieur HUYGENS, to underſtand in what Degree it repreſents things diſtinct and free from colours; and to know the aperture by which it admits light: And after the words [— *Verſus Focum* E *reflectatur.*] (*Pl.* IV. *Fig.* 2.) it may not be amiſs to add this note.

Conſe-

Conferendo diſtantias foci iſtius a verticibus Lentis & ſpeculi concavi, hoc eſt, EF $\frac{1}{6}$ *&* ETU $6\frac{1}{3}$ *dig. prodit ratio* 1 *ad* 38; *qua indicatur objecta* 38 *vicibus circiter ampliari.*

And to this proportion is very conſentaneous the obſervation of the crowns on the weathercock: For the ſcheme repreſents it bigger by $2\frac{1}{2}$ times when ſeene through this then when through an ordinary perſpective. And ſo ſuppoſing that to magnify 13 or 24 times, as by the Deſcription it ſhould, this by the experiment proportionably muſt magnify almoſt as much as I have aſſigned it.

To the objection, that with it objects are difficultly found, I may anſwer, that that's the inconvenience of all Tubes that magnify much; and that after a little Uſe the Inconvenience will grow leſs: For I could readily enough find any day-objects, by knowing which way they were poſited from other objects that I accidentally ſaw in it; but in the night to find Starrs, I confeſs is troubleſome enough. Yet this may be eaſily remidyed, by two ſights affixed to the iron Rod by which the tube is ſuſteined. And ſuch I once intended ſhould have beene made before I ſent it away from mee, but that I thought the defect would not be adjudged material. If ſuch ſights be not found a ſufficient remedy, there may bee an ordinary perſpective glaſs faſtened to the ſame Frame with the tube, and directed to the ſame object; as DES-CARTES in his Dioptrics hath deſcribed for remedying the ſame Inconvenience of his beſt teleſcopes.

The plane ſide of the eye-glaſs is apt to bee ſoiled with duſt falling upon it; and therefore the little leaden Ring put into the orifice of the bigger leaden barrel to moderate its aperture, muſt be ſometimes taken out, and the glaſs wiped with leather done upon the ſmall end of a ſtick, or other ſuch like contrivance; but care muſt bee taken that the ſaid Ring bee not loſt, for without it Objects appear very confuſed at the edges of the apparent ſpace. So if the concave metal contract any dullneſs by moyſture, or otherwiſe, it ought to be taken out and rubbed with gentle leather, but not with Putty, or any thing that may weare the metal.

I am very ſenſible of the Honour done me by the Biſhop of *Sarum*,* in propoſing mee Candidate; and which, I hope, will bee further conferred upon me by my Election into the Society. And if ſo, I ſhall endeavour to teſtify my gratitude, by communicating what my poor and ſolitary endeavours

can

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* Seth Ward, D. D.

can effect towards the promoting your Philoſophical deſigns.

SIR,

I am

Your very humble
Servant,

I. NEWTON.

NUMB. VI.

St. Andrews, 7 *March*, 1673.

SIR,

I Have received yours, dated *Febr.* 20, togither with Mr. NEWTON's Anſwer, with which I am exceedingly ſatisfied. I am much engaged to you both for the Pains ye have been at. I am almoſt convinced that oblique reflection cauſeth more Light then the direct; but I am not fully perſwaded that it is more regular. I conceive that the rudelie poliſhed plate of metall in an oblique poſition cauſeth the image appear more diſtinct, becauſe the obliquitie hideth the concavities, ſo that no rayes come to the eyes but from the tops of the litle *tubercula*; which ar certeinlie beſt poliſhed, the other rayes which confuſed the image being keeped away. But if the plate be exactlie poliſhed (I ſpeak here as to ſenſe) the poſition muſt be ſo oblique, before the inſenſible concavities can be hide, that the plane ſall almoſt turn, to the ſight, in a line. I Grant that I have been miſtaken in that firſt Advantage which I mentioned: for the plane Speculum F having certenlie (as all human artifice hath) ſome errors in it, cauſeth greater prejudice by their being remote from the focus than being near to it; and in it ther is none at all cauſed: wher if it could be placed, and an near and direct aſpect had of it, this wer certeinlie the beſt teleſcope of this Sort.

It is true indeed, that in teleſcopes with convex or concave *Specula* to double the charge, the lenth muſt be almoſt doubled; but to double is a great alteration, and hardlie ſufferable (as I ſuppoſe) in werie good glaſſes, if the leaſt charge be conſiderable. But I underſtand not how the charge can be altered at all with the ſame glaſſes in Mr. NEWTON's Teleſcope; for I know nothing of that which was deſcribed to Mr. OLDENBURGH. It is true that eye-glaſſes can be charged in all Teleſcopes if they be at hand of the required depth. I think there is no great Hazard in theſe teleſcopes of overcharging, ſeing the charge of the Eye-Glaſſe can be diminiſhed at Pleaſur;

ſur; nether upon this account needs the angle of viſion be ſo ſmall, ſeing it is equal to the angle of the eye-glaſſe from its focus, its other focus being the litle ſpeculum; nor the darknes at all augmented, if the apertures of the ſpeculums be proportional to the diameters of the Spheres. But above all things I deſire to know this; that ſeing the Image made by the great Speculum may be eſteemed a ſmal viſibile, and ſeing Mr. NEWTON in the *Tranſactions*, Page 3080, thinketh it fitter to mak an microſcope or tube to behold an ſmal viſibile of one concave Speculum and one eye-glaſſe, rather then with one ſingle Eye-Glaſſe, and much rather than with one plane Speculum and with one Eye-Glaſſe: wherfor alſo to look to this ſmal viſibile, the firſt alſo ſould not be preferred to the laſt. This image indeed is not capable of ſuch magnification as an viſible is; yet I am hardlie ſenſible how this ſould caſt the ballance, taking in the defects of a plane Speculum, togither with other inconveniencies in taking up the object. [I ſaid indeed, that hyperbolick and elliptick Glaſſes wer tryed in vaine; but I ſpoke not ſo of ſpherick Speculums (as Mr. NEWTON's Words ſeem to imply, *Tranſactions*, Page 4059.) for any thing I did deſerves not the Name of a Tryall, ſeing Mr. *Rive* and

Mr. *Cox* both know that the great Speculum was poliſhed onlie with a Cloath and puttie: nether thought I it worth the pains at that time to be ſerious about further enquiry in that buſines; for they undertook indeed to poliſh a leſs Speculum to me upon the tool. I am not yet fullie convinced which of theſe two ways have the advantage; albeit I incline more to Mr. NEWTON's, eſpecialie becauſe of the ſmal diſtance betwixt the plane Speculum's focus and the eye. However, Experience muſt determine all; neither am I concerned how it happen. I had no Intention that my thoughts of theſe teleſcopes ſould be printed; my deſigne was onlie befor, as now, that (if ye thought fitt, otherways not) ye might ſend them to Mr. NEWTON, *&c.* —

Mr. NEWTON's Diſcourſe of Reflection puts me in mind of a Notion I had of burning glaſſes ſeveral pears ago; which appears to me more uſefull than ſubtile. If ther be a concave Speculum of glaſſe, the leaded convex Surface having the ſame Center with the concave (or to ſpeak preciſelie, albeit perchance to little more Purpoſe, let the radius of the convexitie be c, the thicknes of the glaſſe *in axis tranſitu* f, the Radius of the Convexitie equal to $\frac{9c^2 + 18cf + 5f^2}{9c + 5f}$) this Speculum ſal have the *Foci* of both the Surfaces in the ſame Point; and not onlie that, but all the Rays, which ar reflected betwixt the two Surfaces, ſal in their Egreſſe come *quam proximè* to the common Focus. The making of ſuch an Speculum requireth not much more Airt then an ordinar plane Glaſſe, ſeing great Subtilitie is not neceſſar here: So that I believe they, who mak the plane Miroir-Glaſſes, wold mak one of theſe three Foot in Diameter for four or five L. ſt. or litle more: For I have ſeen plane Glaſſes almoſt of that Bignes ſold even here for leſs Money. Now ſeing (as Mr. NEWTON obſerveth) that al reflecting Metalls loſe more then $\frac{1}{3}$ of the Rayes: This concave Glaſſe even *cæteris paribus*, wold have an great Advantage of a Metall one; for certainlie an exactlie poliſhed thin Miroir-Glaſſe of good tranſparent Mater, after a few Reflections, doeth not loſe $\frac{1}{4}$ of the Rayes: And upon other Accounts this hath incomparable Advantages, ſeing it is more portable, free from tarniſhing, and above al hardlie $\frac{1}{20}$ of the Value. The great Uſefulneſs of Burning Concaves, this being ſo obvious, and as yet (for qt. I know) untouched by anie, makes me jealous that there may be in the Practiſe ſome Fallacie. Ye may communicate this to intelligent Perſons, and eſpeciallie to Mr. NEWTON;

aſſuring him, that none hath a greater Veneration for him, admiring more his great and ſubtile Inventions, then his and

Your humble Servant,

J. GREGORIE.

If ye pleaſe, let me hear with the firſt convenience what may be judged the reſult of this burning concave: for I am as much concerned, to be undeceived, if ther be any inſuperable difficultie, as to be informed of an moſt ſurprizing ſucceſs. I have ſpoke of it to ſeverals here, but al wer as ignorant of it as my ſelf, *&c.*

[I deſire yet to be more particular in the mater of teleſcopes. I ſuppoſe an 4 foot teleſcope have the aperture *6* inches; the litle concave having the aperture $\frac{3}{4}$ inches, may magnifie 8 times, the radius being 1 foot. In this caſe the hole in the midle of the great concave is onlie $\frac{1}{4}$ inche, which being fitted with an eye-glaſſe equallie convex on both ſides, amplifying the Charge of the litle concave 24 times, doeth mak an teleſcope magnifying the object 190 times (which is no extraordinar charge, ſeing Mr. NEWTON's table giveth 171, and might be much leſs without inconvenience) taking in an angle of viſion of above 20 Degr. and with this ther is not loſt $\frac{1}{60}$ of the rays. with the Loſſe of $\frac{1}{36}$ of the Rayes it might magnifie not above 144 times, and tak in an angle of viſion of above 28 Deg. with al this the midle of the object is illuſtrat with all the rays which the aperture of the great concave doth reflect. by theſe means I think that I keep of from theſe two inconveniences mentioned by Mr. NEWTON in the ſeventh particular of his conſiderations. the event of theſe other conſiderations, as I ſuppoſe, can onlie be determined exactlie by experience.]

NUMB. VII.

SIR,

HAving peruſed Mr. GREGORY's candid reply, I have thought good to ſend you theſe further conſiderations upon the differences that ſtill are between us. And firſt, that a well poliſhed plate reflects at the obliquity of 45 degrees more truly than direct ones, ſeems to me very certain: for the flat *tubercula*, or ſhallow valleys, ſuch as may be the remains of ſcratches almoſt worn out, will cauſe the leaſt errors in the obliqueſt rays which fall on all ſides the hill, excepting on the middle of the foreſide and backſide of it; that is, where the hill inclines directly towards or directly from the ray: for if the ray fall on that ſection of the hill, its error is in all obliquities juſt double to the hill's declivity: but if it fall on any other part of the hill, its error is leſs than double, if it be an oblique ray, and that ſo much the leſs, by how much the ray is obliquer; but if it be a direct ray, its error is juſt double to the declivity, and therefore greater in that caſe. I preſume Mr. GREGORY, if you think it convenient to tranſmit this to him, will eaſily apprehend me.

How the charge may be varied at Pleaſure in my teleſcope, will appear by this Figure; where A repreſents the great concave, E the Eye-Glaſs, and BCD a Priſm of Glaſs or Cryſtal, whoſe ſides BC and BD are not flat, but ſpherically convex; ſo that the rays which come from G, the focus of the great concave A, may, by the refraction of the firſt ſide BC, be reduced into parallеliſm, and, after reflexion from the baſe CD, be made by the refraction of the next ſide BD, to converge to the focus of the eye-glaſs H.

[*Plate* 4, *Fig.* 8.

The Teleſcope being thus formed, it appears how the charge may be altered, by varying the diſtances of the glaſſes and ſpeculum.

As for the Objection, That Mr. GREGORY's Teleſcope will be either overcharged, or have too ſmall an angle of viſion, *&c.* I apprehend that the difference between us lies in limiting the Aperture of the eye-glaſs. Mr. GREGORY puts it equal to that of the little concave; but I ſhould rather determine it by this proportion; That if a middle point be taken between the Eye-glaſs and its focus, the apertures of the eye-glaſs and concave will be proportional to their Diſtances from that Point: That is, ſuppoſe A B the little concave, E F the eye-glaſs, G H their common focus or image, and K ther mean diſtance between G H and E F; from the extremities of A B draw A K and B K, butting on the Eye-Glaſs at F and E, and E F ſhall be its Aperture. The reaſon of this limitation is, that the ſuperfluous light which comes on all ſides of the ſpeculum A B to the ſpace G H, in which the picture of the object is made, may fall beſides the eye-glaſs: For if it ſhould paſs through it to the eye, it would exceedingly blend thoſe parts of the picture with which 'tis mixed; and ſuch are thoſe parts of it which extend themſelves beyond the lines A K, B K. As I remember, I ſaid in my former

Plate 4, *Fig.* 10.]

former letter, that the ſcattering Light which falls on the eye-glaſs will diſturb the viſion; and this is to be underſtood of any ſtraggling light which comes not from the picture; but if it come from the picture to the eye-glaſs, the diſturbance will be much greater, ſo as not to be allowed of. Againſt the firſt, I ſee no very convenient remedy; and againſt the laſt, none but aſſigning a ſmall Aperture to the eye-glaſs; ſuppoſing the Teleſcope is uſed in the Day-time, or in twilight, or to view the Moon, or any ſtarr very neare her, or neare the brighter Planets. And if for this reaſon the Aperture be limited by any rule, the angle of viſion will become very ſmall, as I affirmed: For Inſtance, in that caſe where Mr. GREGORY in his Poſtſcript puts it above 20 degrees, it will be reduced to leſs than half a degree. Yet I confeſs there is a way by which the angle of viſion may be ſomething enlarged; but it will not be very conſiderable, unleſs the eye-glaſs be alſo deeper charged.

Why I aſſign a concave with an eye-glaſs to magnify ſmall Objects (in *Tranſactions*, Page 3080.) and yet an eye-glaſs without ſuch a concave to magnify the image of the great concave, which is equivalent to a ſmall object, is, becauſe that image doth not require to be magnified ſo much as an object by a Microſcope; and further, becauſe the angle of the penicil of rays which flow from any point of the ſmall object, that the object may appear ſufficiently luminous, ought to be as great as poſſible; and a concave will with equal diſtinctneſs reflect the rays with a greater angle of the penicill than a Lens; but in the Teleſcope the Angles of thoſe Pencils are not ſo great as to tranſcend the limits at which an eye-glaſs may with ſufficient diſtinctneſs refract them: And therefore in theſe inſtruments I choſe to lay all the ſtreſs of magnifying upon the eye-glaſſes. In Microſcopes alſo I would lay as much ſtreſs of magnifying upon the eye-glaſs as it is well capable of, and the exceſs only upon the Concave.

Concerning my citation of Mr. GREGORY againſt Monſ. CASSEGRAIN, the force of it lies only in the inference that Optic Inſtruments moſt probably, according to M. CASSEGRAIN's deſign, have been tried by Reflexion; which I think I might well infer without having regard to the ſpecific figure of the ſpeculum which Mr. GREGORY there ſpake of: And therefore I think it cannot be ſaid that I made him ſpeak of ſpheric figures, where his meaning was of hyperbolic and elliptic ones. But if I ſhould be ſo underſtood, becauſe I put the figure of the great concave to be ſphericall wherever I ſpecify it, I know not why I might not by way of Conſequence make that Interpretation: For it is not probable that any man would attempt Hyperbolicke and Elliptic Figures of Speculums, until the Event of ſphericall ones had been firſt tried.

And accordingly the tryal of Mr. GREGORY with Mr. *Reive* was by a ſpherical Figure: Which tryal, although I am now ſatisfied that it was made very rudely, yet by the Informaton which I had of it when I wrote the letter about Mr. CASSEGRAIN's deſign, I apprehended it to have been made with very great diligence and curioſity, as I ſignified in my former letter at large. And this I hope may excuſe me for ſpeaking of it in the *Tranſactions* as if it had been tried with more accuracy than really it was. And thus much concerning the Teleſcope.

The deſign of the burning Speculum appears to me very plauſible, and worthy of being put in practice. What Artiſts may think of it I know not; but the greateſt difficulty in the practice that occurs to me is, to proportion the two ſurfaces ſo, that the force of both may be in the ſame point according to the Theory. But perhaps it is not neceſſary to be ſo curious; for it ſeems to me that the effect would ſcarce be ſenſibly leſs, if both ſides ſhould be ground to the concave and gage of the ſame tool, *&c.*

Your humble Servant,

Cambridge, April 9,
1673.

J. NEWTON.

NUMB. VIII.

St. Andrews, 13 *May*, 1673.

SIR,

I Received lately your's, dated the 19th *April*, togither with Mr. NEWTON's to you; for whoſe faire correſpondence I give you Both hearty Thankes: To which I have onlie theſe few things to ſay. As to his firſt, I underſtand not well his meaning: an oblique poſition ſeemeth to expoſe al its inequalities more fullie to the rayes; and ether altogither to hide the loweſt of the regular Surfaces, or otherways to reflect the Rayes coming from them on the adjacent *tubercula*.

His way of varying the charge is indeed exceedinglie ingenious; but I think thoſe Surfaces too lyable to the errors of the artificer's Hand. The opacitie of the glaſſe priſme, togither alſo with the iregularitie which he hath diſcovered in Refraction, may help to darken and confuſe the Sight.

As for the next, I know not if it be worthie of the paines to look with excellent Teleſcopes on terreſtrial bodies: For as the Object is magnified, ſo is the groſſenes of our Atmoſpher to our ſenſe encreaſed; ſo that the one hindereth as the other helpeth. In celeſtial Obſervations any little thing applyed to one or more ſides of the litle Speculum, may ſtape the Rayes of the Moon, or anie other of the brighter Planets, if theſe be alſo thought worthie noticing. I ſuppoſe that al theſe adventitious Rayes may be hindered even in Day-light, by puting in the Focus of the eye-glaſſe towards the eye an thin Plate of ſome Metall with an litle round Hole in the midle in Diameter $\frac{1}{12}$, $\frac{1}{20}$, or $\frac{1}{30}$ of an Inch; which is calculate ſo, as the Diſtance of the Eye-Glaſſe from the litle Concave is to the Diſtance of the Eye-Glaſe from its Focus, ſo is the Aperture of the litle Concave to the Diameter of this Hole. It is true, at ſome times this may hinder ſome of the Rayes, but they ar always the worſt; and by encreaſing the Aperture of the litle Concave not much above what my methode requirs, it will hinder non at all. I could not have judged that Mr. NEWTON had thought on this Inconvenience in Mr. CASSEGRAINE his

Teleſcope, ſeing it ſeemeth to me, even in his own Microſcope, *Tranſactions*, Page 2080. for not onlie the direct Rayes of the Object O (neverthele ſs that it be looked to onlie with Day-light) but alſo theſe proceeding from the Objects befor the Concave ar always ſcattered through the whol Image; nether doe I ſee how it can be exactlie helped.

That we may ſe what Effect this ſcattered Light may cauſe in the Sight, let us ſuppoſe the Teleſcope to magnify 160 times, and the Aperture of the great Concave to contein 8 times the Aperture of the Eye-Glaſſe or litle Concave, and the Object to be a Planet apparent Diameter $\frac{1}{2}$ of a Minute, in whoſe Image ther paſſeth the Rayes of another Planet of the ſame apparent Bignes and Brightnes. The Angle of Viſion is about 16 Deg. the Planet appears in an Angle of $1\frac{1}{3}$ Deg. that is to ſay, it illuſtrats ſo much of the *Retina*. Now the other Planet illuſtrats (I take no Notice of the litle Concave, which is to my Diſadvantage, ſeiag it keepeth of manie of theſe Rayes) 16 Deg. of the *Retina*. Now becauſe of the Aperturs, ther ar 64 times as manie Rayes in $1\frac{1}{3}$ of a Deg. as in 16; that is to ſay, theſe adventitious Rayes have but $\frac{1}{9216}$ of the Splendour of the Image; which I think hardlie ſenſible. The Brightnes indeed of the Moon wer near $\frac{1}{2}$ and

$\frac{1}{2}$ and not ſufferable ; which therfor is to be helped be ſome of the foreſaid Means. In the Twylight the Inconveniencie may be for the moſt pairt verie inconſiderable, and perchance ſomtimes (as alſo other adventitious Rayes) advantagious by making inſenſible the Circumradiancie of celeſtial Bodies. All this is ſuppoſing the Eye-Glaſſe convex ; for if it be concave, the Effect is otherways.

As to his laſt, I imagine that all Images doe require (*cæteris paribus*) to be magnified as much as may be. Nether doeth his other Reaſon appear to me ; for Penicills of the ſame Angles ar more trulie reflected by an Concave, then refracted by a Lens. And albeit in Teleſcopes the ſaid Angle tranſcend not the Limite of a Lens commonlie aſſigned, yet ſurelie the more it is exceeded by this Limite, it is ſo much the better. And al this is obſerved in my Deſigne ; yea ther is 3 times as much Streſs of Magnifying alſo laid upon the Eye-Glaſſe as on the litle Concave. It may alſo be noticed, that here ther ar no verie ſmal Sizes of Spheres to be poliſhed ; which can hardlie be done (as I ſuppoſe) to Preciſenes. It is poſſible that even in Teleſcopes ther may be more ſtreſs laid on the Eye-Glaſs then it can carrie ; eſpecialie in the extreme Penicills, wher the Incidence is oblique, and Refraction perhaps ſo great, that $\frac{1}{25}$ of it may be ſenſible. Alſo an ordinary Microſcope ſuffers no Aperture above the Limite of a Lens; and neverthelefs it doeth much more then one ſimple Lens, or elſe the Worlde hath been exceedinglie deceived. I dare not confidentlie affirme, that ordinary Microſcopes may outdoe any Improvment of one Lens; but if they doe, I think it more then an probable Argument, that my Project ſall exceed Mr. NEWTON's; ſeing beſide the onlie Diſadvantage which I ſee in mien (to wit, the Diſtance of the Glaſſes) it hath the great Irregularitie of Refraction.

I think nothing can be inferred concerning the Tryal of my Teleſcope from my Aſſertion, ſeing the Tryal was after that Aſſertion; but Mr. NEWTON could not be ſuppoſed to know this.

SIR,

Your moſt humbe Servant,

J. GREGORIE.

FINIS.

TO
The Reverend Dr. DESAGULIERS.

SIR,

I Have incloſed your Papers, (which were left with me laſt Week;) what I have added, you will be pleaſed to make uſe of, or not, as you think fit. The firſt is a Rule for the Parts of the *Gregorian* Teleſcope, of the ſame Nature with what I formerly delivered to Mr. *Molineux*, relating to that of Mr. *Caſſegrain*. I have carried the Tables for thoſe with two Eye-glaſſes, both for Night and Day, as far as it ſeems probable to be of any Uſe. I am,

SIR,

Your moſt humble Servant,

Dec. 9th,
1734.

J. HADLEY.

P. S. I have juſt received the Favour of yours. I imagined, the Plate with the ſmall Hole to exclude foreign Light in Teleſcopes, to have been *my own firſt thought*, but find Mr. *Ja. Gregory* had had *the ſame* before.

The Proportions for the ſeveral parts of a Catadioptric Teleſcope of the Form propoſed by Mr. JAMES GREGORY. PLATE 4. Fig. 8.

LET, A D repreſent the common Axis of the Teleſcope, and 2 concave Specula B B and F F. Suppoſe A G the focal length of the Speculum B B, whoſe proper Aperture BB, and Charge, are likewiſe known. Let C C be the Breadth of the Perforation. F F the Breadth of the ſmaller Speculum equal to, or a little greater than C C. I its Focus; N the Eye-glaſs, N A its focal Length, and M a Plate, with a ſmall hole to exclude all foreign Light. And let it be required to take in at one View ſo much of the Object as may appear through the Teleſcope under a given Angle, *viz.* = C N C. To do this with the loſs of the feweſt Rays of Light near the Axis, the Proportions ſhould be as follow.

Call A G a.
B B b.
The Power or Charge m.

The Ratio of twice the ſemitangent of the apparent Angle of Comprehenſion required C N C, to Radius *i. e.* $\frac{CC}{AN} = n$.

Then H H the Breadth of the Image of ſo much of the Object as is ſeen at once, will be $= \frac{n}{m} a$.

[N. B. *If inſtead of* $\frac{n}{m}a$, *you ſubſtitute c for* HH, *the Algebraic Expreſſions become ſomething more ſimple, for which reaſon I have added them.*]

The Breadth of the Perforation CC; of the great Speculum $= \frac{na + \sqrt{na} \times \sqrt{na + mb}}{m}$, or $c + \sqrt{bc + cc}$.

The focal Length of the ſmall Concave ID.

$$\frac{a \times \sqrt{na} \times \overline{\sqrt{na} + \sqrt{na + mb}}}{\overline{na + mb} + 2\sqrt{\overline{na} \times \overline{na + mb}}}$$

or $\frac{a \times \overline{c + \sqrt{bc + cc}}}{b + c + 2\sqrt{bc + cc}}$.

The Diſtances of the Specula, *i. e.* AD.

$a + \frac{a\sqrt{na}}{\sqrt{na + mb}}$, or $a + \frac{a\sqrt{c}}{\sqrt{b + c}}$.

The focal Length of the Eye-glaſs, and its Diſtance from A, *i. e.* AN.

$\frac{a}{m} + \frac{\sqrt{naa + mab}}{m\sqrt{n}}$, or $\frac{c + \sqrt{bc + cc}}{n}$.

The Diſtance of the Plate M, behind the Eye-glaſs NM, $= \frac{DN \times AN}{DA}$.

The Breadth of the Hole in M $= \frac{NM \times CC}{DN}$.

If a double Eye-glaſs be uſed with this *Teleſcope*, to prevent the Objects being coloured near the Edges of the *Area,* the Image of the Object muſt be thrown back by the ſmaller Concave, ſo far behind the great *Speculum*, that there may be room enough to place the firſt Eye-glaſs N at a ſufficient Diſtance before it, and the Algebraic Expreſſions of the ſeveral Parts become much more complex, wherefore I have omitted them, and added the Proportions for the following Sizes.

For the Night.

If A G be	40 inches,	60	90
and Power	172.	234.	317.
BB	4.9	6.⅔	9.0
ID	4.28	5.88	8.01
FF	0.67	0.81	0.97
CC	0.66	0.80	0.96
focus N	4.23	5.15	6.29
length S	1.52	1.82	2.2
AD	44.72	66.4	98.68
AN	0.9	1.1	1.⅛
NS	3.04	3.64	4.4
SM	0.8	0.93	1.13
RR	0.43	0.52	0.63

For the Day.

If A G be 40 inches, and the Power 86.

BB = 4.9 AD = 6.74
ID = 5.95 AN = 0.9
FF = 1.0 NS = 4.44
CC = 0.99 SM = 1.15
N = 6.02 RR = 0.63
S = 2.22

FINIS.

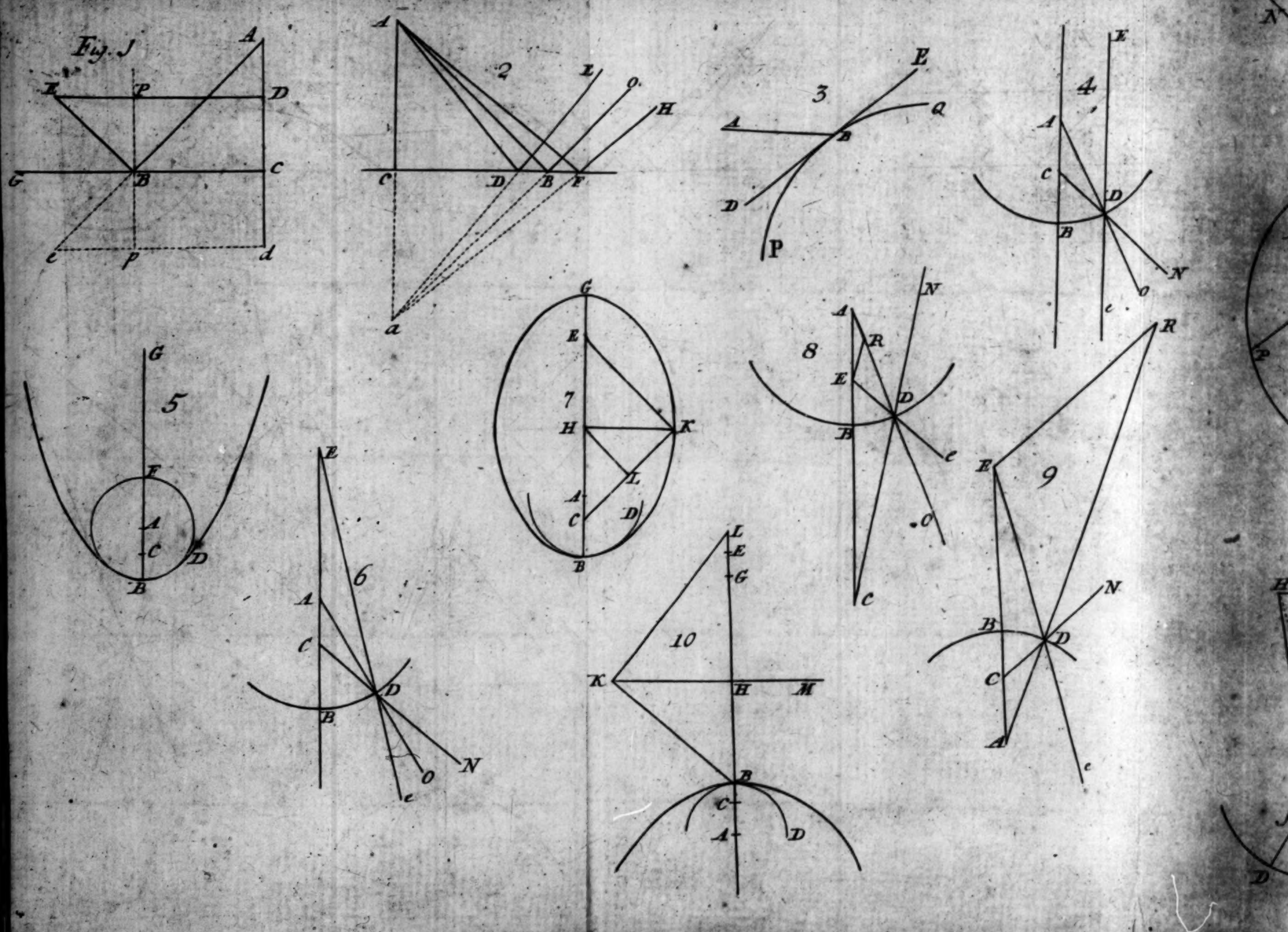
Fig. 1
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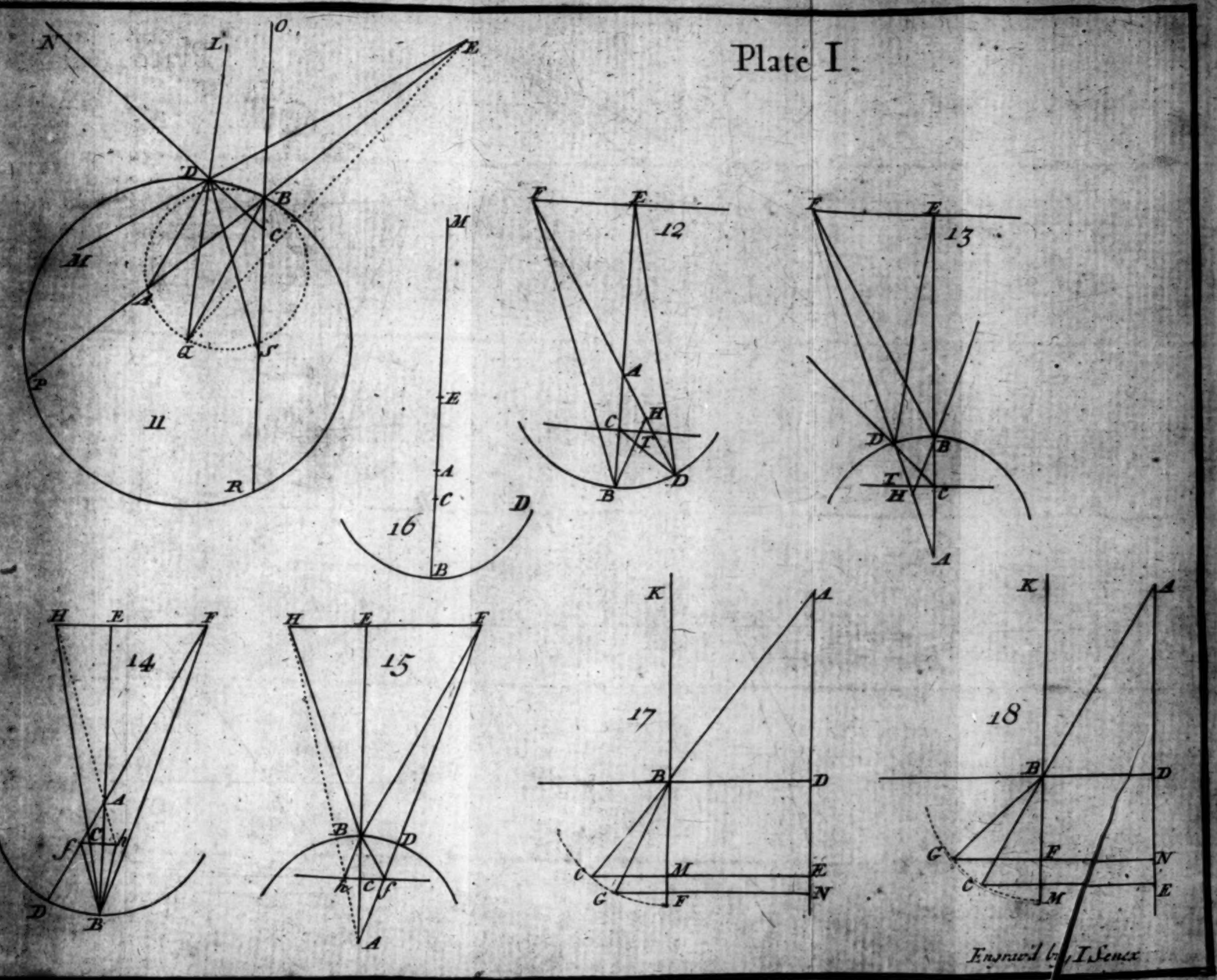
Plate I.



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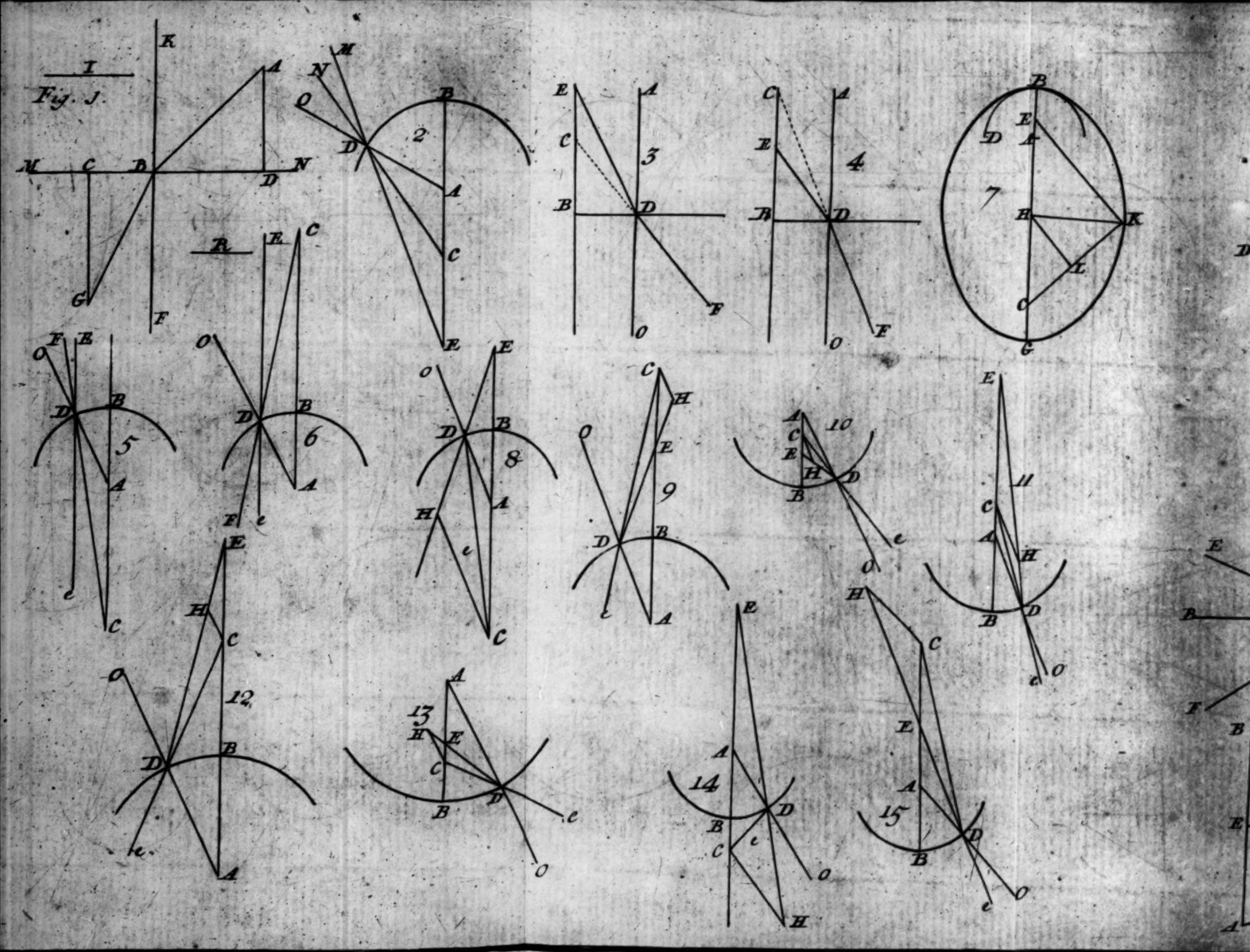
Fig. 1
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Plate II

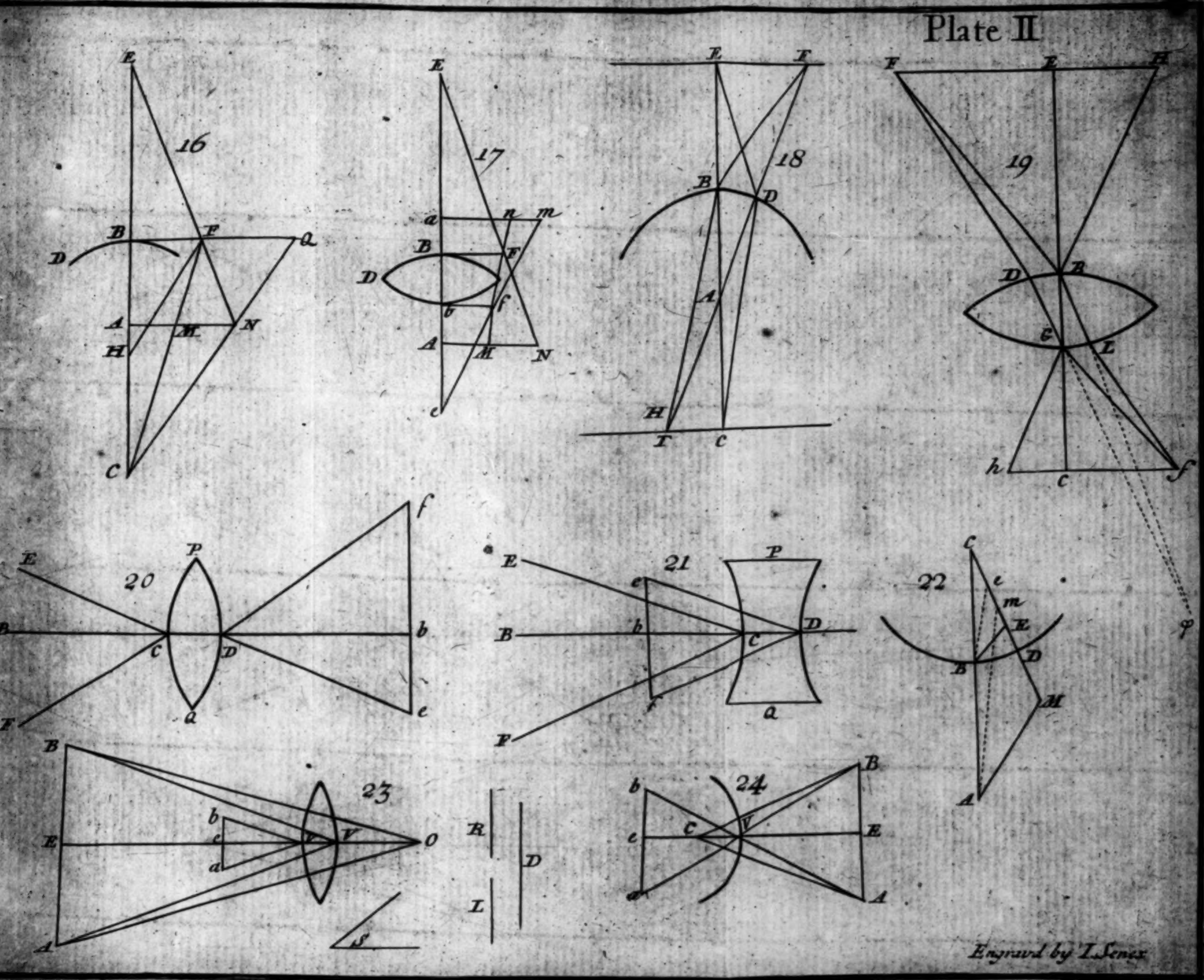

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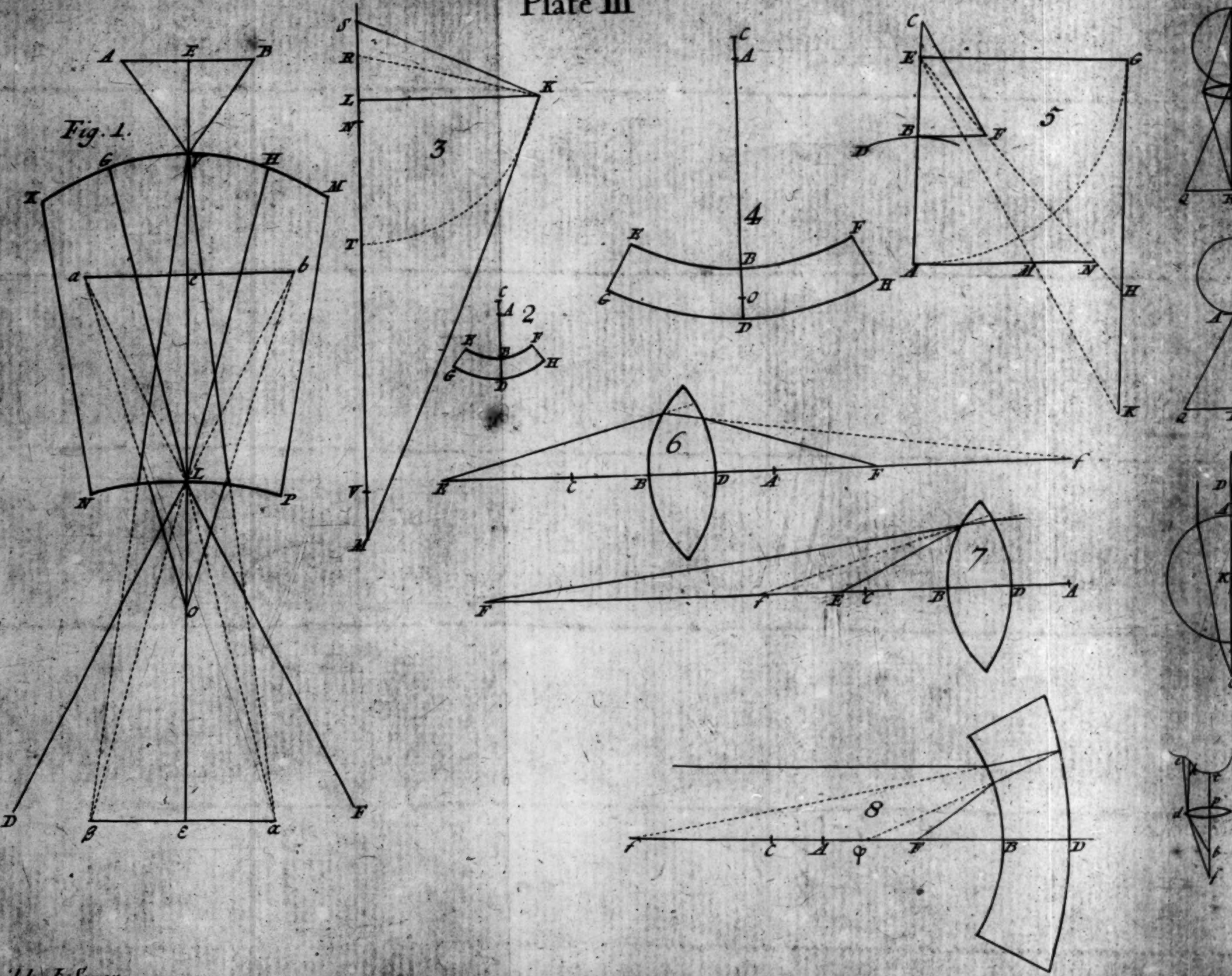
Plate III



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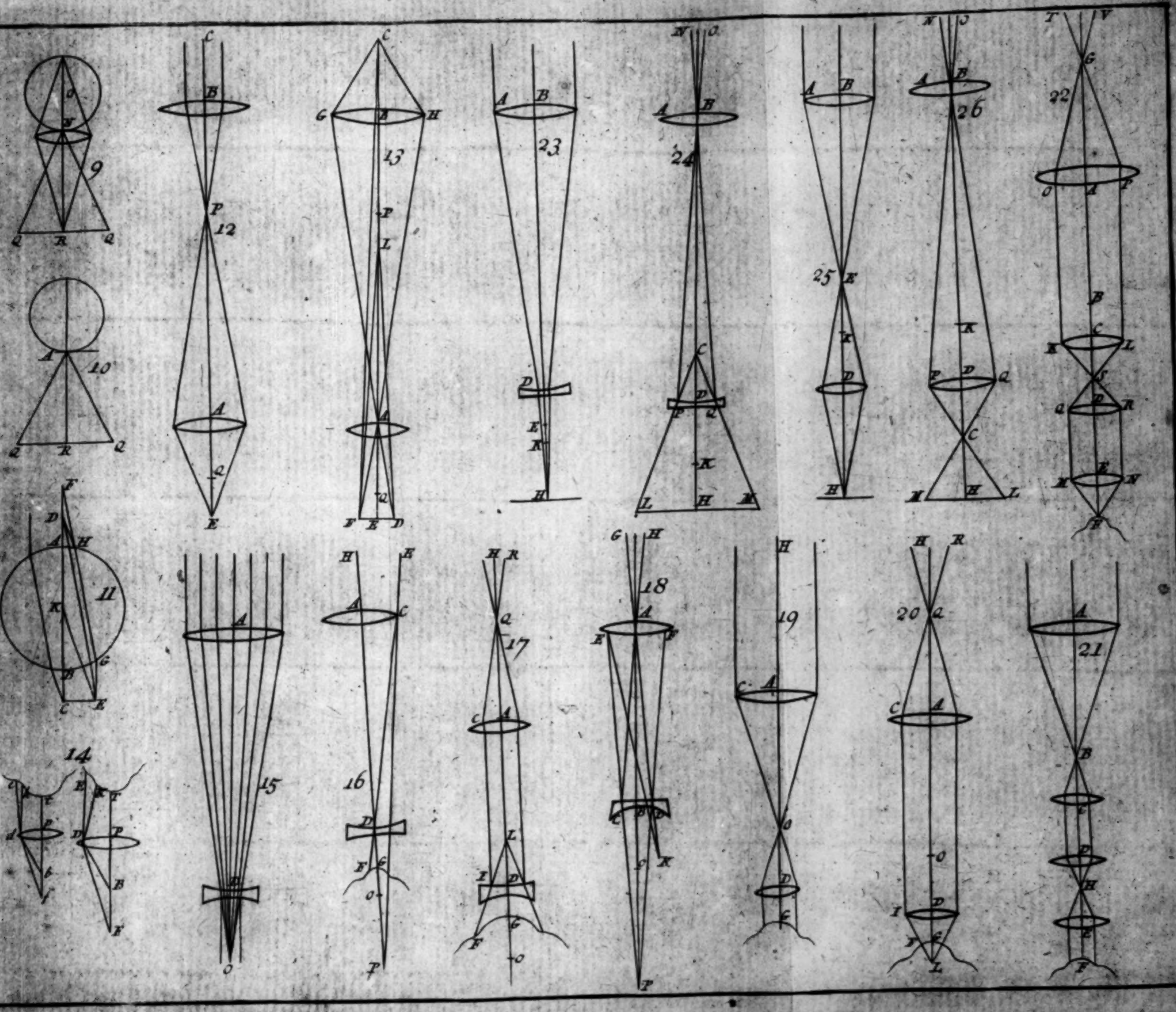

9
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c k t d p b f
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C B P A Q E
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C G B H P L A Q F E D
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A D O
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H E A C D F G O P
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H R Q C A L I D F G O
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G H A E F C B D K P
19
H C A B D G
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H R Q C A O I D F G L
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A B C D H E F
23
A B D E K H
24
N O A B C P D Q K L H M
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A B E K D H
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N O A B K P D Q C M H L
22
T V G O A P B C K L S Q D R E M N F

Plate. IV.

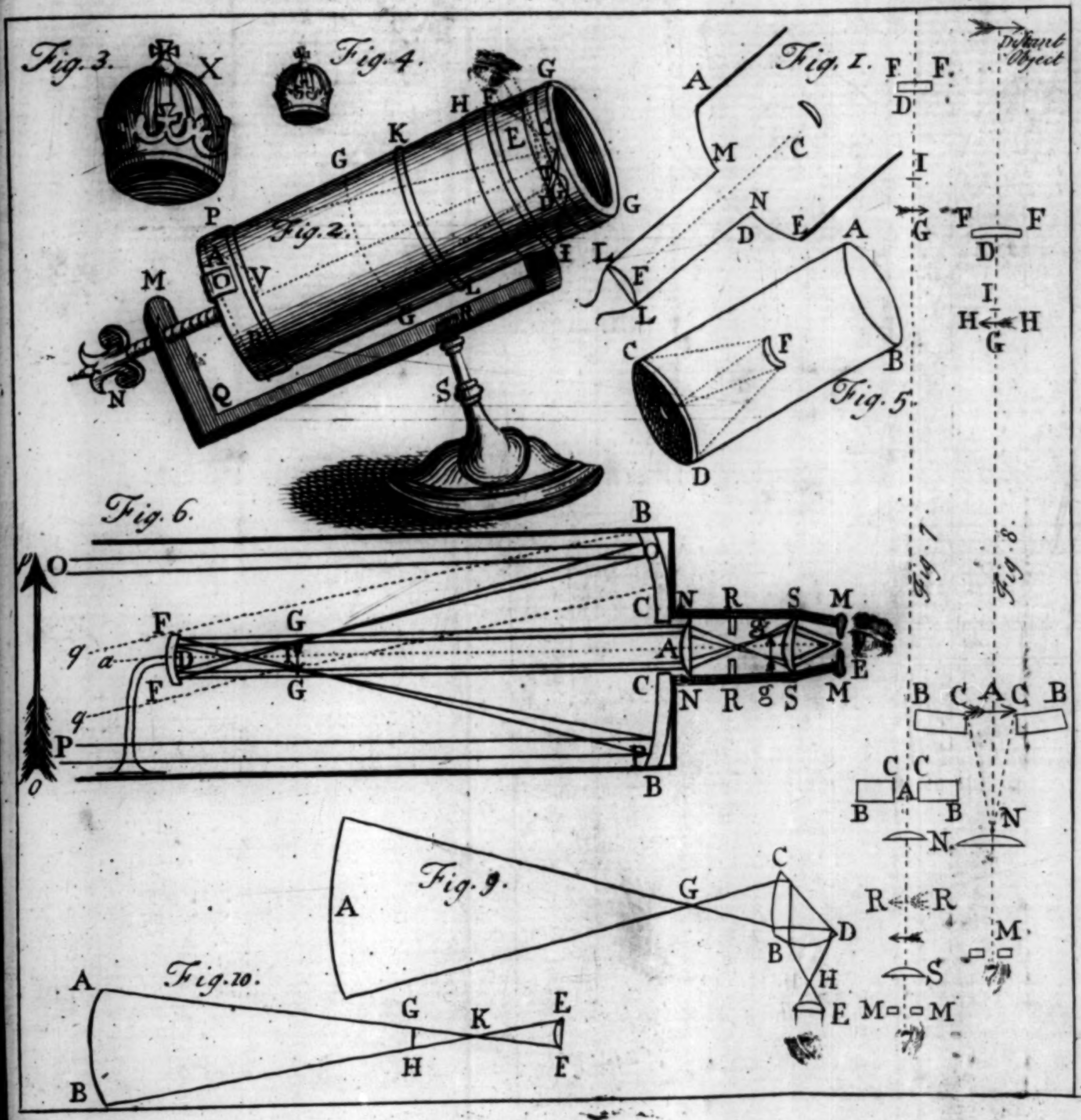
Fig. 1
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Fig. 2
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Fig. 3
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Fig. 4
Fig. 5
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Fig. 6
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Fig. 7
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Fig. 8
Distant Object
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N
M
Fig. 9
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Fig. 10
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Errata & Corrigenda.

Pag.	Line	For	Read
11	23	A B C	*p* B *c*
60	2	$- 2dx \times 2xx\ \mathrm{I} \times 2xx$	$\mp 2dx \pm 2xx\ \mathrm{I} \times 2xx$
93	22	$\mp 6arz \pm 2cvz \mp 4acr$	$\pm 6arz + 2crz \pm 4acr$
ibid.	27	$\frac{2ab + 2ar}{a + b}$	$\frac{2ab \pm 2ar}{a \pm b}$
117	2	after *collected*, inſert,	*or diverge as if they were collected.*
ibid.	9	dele *from*, whence, *to* therein,	
124	19	$x = \frac{dr}{2d + r}$	$r = \frac{dx}{d + x}$
128	4	Concave	Convex.
131	19	$d = \frac{b - cr}{2c}$	$d = \frac{br - cr}{2c}$
133	6	$: : 2d + rr ;$	$: : 2d + r : r ;$
ibid.	7	$: : r = 2d . r .$	$: : r - 2d : r .$
139	20	$+ \frac{1}{2} rst$	$+ \frac{4}{3} rst$
140	17	after *is*, inſert	$\frac{6 . drr}{6 . dr}$, or.
143	*ult.*	$+ \mathrm{I} + s\mathrm{R}t$	$\mathrm{I}s + \mathrm{R}t.$
146	13	*or*	*and.*
147	24	$4rst + 64py - 2rty.$	$4rst + 6rsy - 2rty.$
149	8	after *have* inſert	$d = \frac{-6rrr,}{6rr - 6rr}$ or.
151	3	$\frac{\mathrm{I}}{\mathrm{I} - \mathrm{R}}$	$\frac{\mathrm{R}}{\mathrm{I} - \mathrm{R}}$
164	9	*refracted,*	*reflected,*
165	14	B*fn*, the ſecond D*φx*,	B*f*,*u*, the ſecond D*φ*,*x*,
166	3	$6a + 2c - r$	$6a + 2c - \mathrm{Z}$
168	10	$\frac{15}{268}$	$\frac{15}{848}$
ibid.	18	$1 \frac{31}{48} a$	$1 \frac{37}{48} a$
ibid.	19	$\frac{1}{25} a$	$\frac{1}{11} a$
170	3	after Prop. 1. refer to	*Plate* III.
180	1	after Prop. 2. refer to	*Plate* III.
183	9	after *Microſcopes*, refer to	*Plate* III.
187	2	after Prop. 3. refer to	*Plate* III.
194	8	after Prop. 4. refer to	*Plate* III.
196	15	E L	D L
197	23	after Prop. 5. refer to	*Plate* III.

Pag.

Errata & Corrigenda.

Pag.	Line	For	Read
202	15	*leſs*	*more.*
206	1	after Prop. *6.* refer to	*Plate* III.
207	8	*Lens's*	*Lenſes.*

N. B. For want of Room, for the *ſixth* Figure, (in Plate IV.) the Rays O O and P P ſeem parallel to one another; but they muſt be ſuppoſed to have croſs'd at the Center of the *Speculum* upon the *Axis* A *a* produc'd, as is repreſented by the Rays *q* O, *q* C, inſtead of O O; for the Rays O O, do really come from the Bottom of the *diſtant* Object *p o*, at *o*, and the Rays P P from the Top of the ſaid Object at *p*.

Theſe Errata & Corrigenda *render this Work compleat.*

J. T. D.